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UNITED STATES COAST SURVEY:

CARLILE P. PATTERSON, SUPERINTENDENT.

METHODS, DISCUSSIONS, AND RESULTS.

FIELD-WORK OF THE TRIANGULATION.

WASHINGTON:
GOVERNMENT PRINTING OFFICE.
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U. S. Coast Survey.

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ERRATA.

Page 1, line 6, for "3 4" read "3-4."
Page 1, line 34, for "14" read "14-15."
Page 1, line 50, for "21" read "20-21."
Page 5, line 8, for "enths" read "tenths."
Page 17, near bottom, for "Schott" read "C. A. Schott."
Page 23, line 3 of text, for "concluded" read "c, for concluded."
Page 23, lines 4 and 5 of text, for "Greylock, (Mount Rafinesque,") read "Greylock to Mount Rafinesque."
Page 24, second line of third triangle, column of spherical angles, omit "52. 77."
Page 24, third line of third triangle, column of spherical angles, insert "15. 80."
Page 24, second line of fourth triangle, column of spherical angles, for "40. 36" read "40. 34."
Page 36, line 9, for "Schott" read "C. A. Schott."

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FIELD-WORK OF THE TRIANGULATION.

By Richard D. Cutts, Assistant.

[Reprinted, with additions, from the Coast Survey Report of 1868.]

Geodesy, in practice, may be described as a system of the most exact land-measurements, extended, in the form of a triangulation, over a large area; controlled, in its relation to the meridian, by astronomical azimuths; computed by formulæ based on the dimensions of the spheroid; and placed in its true position on the surface of the earth by astronomical latitudes and differences of longitude from an established meridian.

In inverse order the same system of operations, when conducted in the general direction of the meridian or of the parallel, determines the length, in standard measures, of the astronomical degrees comprised within the measured arc, and hence redetermines a value for the dimensions and figure of the earth.

Three orders of triangulation are recognized in the geodetic operations carried on for the survey of the coast:

I. The primary series, with sides varying in length from 20 to 146 miles, such as those which are completed or in progress from the river St. Croix to Cape Henry; between the Kent Island and Atlanta bases; on the line of the geodetic connection between the Atlantic and Pacific coasts; and from Oregon to San Diego.

II. The secondary series, with sides from 5 to 40 miles in length, either connecting the tertiary with the primary, or which, starting from independent bases, is being gradually extended over the coasts, sounds, and bays from the mouth of the Chesapeake to the Rio Grande; and, on the Pacific, from British Columbia to the southern boundary of California.

III. The tertiary triangulation, with sides less than 6 miles in length, which follows the immediate line of the coast for the use of the topographical and sounding parties, and includes the short series which branch off from the secondary and are carried up the smaller rivers and inlets of the sea.

The primary and part of the secondary series, composed principally of quadrilaterals, and verified, at intervals, by the measurement of additional bases and new determinations of the astronomical azimuth, latitude and longitude, constitute the standard geodesy of the survey. From this special class of the geodetic work, subject to the least probable error, the dimensions and figure of the earth are deduced, as in the measurements, made and in progress, of the arcs of the meridian in the Eastern, Middle, and Southern States, and of the arc of the thirty-ninth parallel across the continent.

RECONNAISSANCE.

A system of simple triangles entails the least labor; one of hexagons covers the largest area; while a series of quadrilaterals secures the greatest degree of accuracy. All three of these systems are employed in the survey, according to the necessities of the case, or to the particular object to be accomplished, or to the general rule to obtain results of the highest standard.

A careful reconnaissance invariably precedes the selection of new points for the continuation of the geodetic work of the survey. The first step will be to decide upon the proportions of the scheme best adapted to the character of the country and for the success and progress of the work, and the next the reconnaissance in detail. In the case of high elevations and an open country, little difficulty will be encountered; but if the hills are densely wooded and tolerably uniform in height, the greatest care and skill are needed to select such intervisible points as are the most favorably situated, not merely for the extension of the triangulation, but to satisfy other conditions imposed by the survey. Should the land be uniformly low and clear, the triangulation may be laid

off, as on paper, restricted only in its proportions by the curvature of the earth and the height to which the signal and instrument should be elevated; but if covered with forest or heavily-timbered swamp, the length of the lines will be governed by the labor and expense of opening them, taking into view the possibility of carrying on a smaller series of triangles immediately on the coast, or of a direct measurement of the beach, to be continued until such difficult section has been passed.

If the reconnaissance covers any extended portion of the coast, and a scheme be adopted for the geodetic work, the question will arise whether the proposed triangulation, from its proportions, as determined by the character of the country, will need verification before joining on with the principal bases as measured in each section; and if so, at what intermediate points could such subsidiary lines and azimuths be measured and observed for the correction of the distances and directions.

In the performance of the above duty, the assistant will keep steadily in view the requirements alike of the triangulation and of the survey; and it will be his aim so to modify and adjust them, each in the ratio of its value, to the special features of the country under examination, as to produce a plan of triangulation which, while it satisfies the conditions prescribed, will be the most effective in its results and economical in its execution. The most important of these requirements, beyond the paramount condition of the certain intervisibility of the stations intended to be connected, are

I. The adoption of the highest elevations.

II. The maximum length of line consistent with the limit of 30° prescribed for the least size or an angle. A smaller angle is admissible in quadrilaterals, and also at one end of a base of known line, but not at any new point to be determined.

III. The forming of quadrilaterals whenever possible.

IV. The modifications or changes which can be effected in the position of the proposed stations, so as to avoid, as much as possible, the labor and expense of opening lines through the forest or swamp.

V. The sweep of the horizon, or of the area to be surveyed, with a view to the easy determination of intermediate stations, and of light-houses, spires, chimneys, or other prominent objects not more than two or three miles apart, for the special use of the plane-table and hydrographical parties.

VI. The capacity of the station ground to be protected from the destructive effects of storms and waves and from the ordinary pursuits of man, with a view to the preservation of the station for future use.

VII. The consideration of the altitude to which the theodolite must be raised to escape the variable refraction incident to the visual ray passing close to the surface of the ground.

In case a base of verification is required within the area covered by the reconnaissance, the examination will be more in detail. For the primary work, its length should be not less than 6 miles, and for the secondary between 2 and 3 miles. The site selected should be a level piece of land, the slope of the ground not to exceed four degrees; it should be as free as possible of accidents, such as rivulets, ravines, or irregularities in the surface, in order that the preparation of the line may not be too expensive; and its termini should be in such positions that they can be connected by well-shaped quadrilaterals, either directly or through a short series, with the main triangulation in its vicinity.

The reconnaissance requires the following outfit: A telescope and tripod, the latter with a small circle read by a vernier to 20 minutes; a declination-needle, to be clamped, at will, on the telescope, so that the zero-mark and the visual axis of the telescope will be in the same line; and, if in a hilly or mountainous country, a pocket aneroid barometer and detached thermometer. The index and scale errors, if any, of the aneroid should be determined and entered in the record. If of small value they can be overlooked, as differences and not absolute heights are measured.

The following formula will give the difference in height between two stations at which the aneroid and thermometer have been read within a few hours' interval of time. The shorter the interval, the better.

$$\text{Difference} \ldots\ldots = 60345 \text{ feet } (\log B - \log b)$$

$$\text{Mean temperature} \ldots\ldots = \frac{T+t}{2}$$

The factor varies with the mean temperature. For every degree above 32° F., add 134 to 60345.

Example.

B = 28.72	log B = 1.4581844	
b = 27.14	log b = 1.4336098	
	0.0245746	log's. 8.3904865
$\frac{T+t}{2}$ = 62°.5, and, hence,	64432	4.8091016
... Difference = 1583.4 feet		3.1995881

Or the difference in height may be taken out from the following table, by interpolation of enths, as follows:

28.72 at 62°.5 for 1220.0 feet
27.14 at 62°.5 for 2803.6 feet

Difference.. 1583.6 feet

Difference in height, in feet, between two stations, at the mean of the two observed temperatures.

Barometer.	Mean of observed temperatures in degrees Fahrenheit.						
	32°.	42°.	52°.	62°.	72°.	82°.	92°.
30.0	.....	.....	.....	.....	.....	.....	.....
29.9	87.5	89.4	91.4	93.3	95.3	97.2	99.2
29.8	175.3	179.2	183.1	187.0	190.9	194.8	198.7
29.7	263.4	269.3	275.1	280.9	286.8	292.7	298.5
29.6	351.8	359.6	367.4	375.2	383.0	390.9	398.7
29.5	440.5	450.3	460.0	469.8	479.6	489.4	499.2
29.4	529.5	541.3	553.0	564.7	576.5	588.2	600.1
29.3	618.8	632.6	646.3	659.9	673.7	687.4	701.3
29.2	708.4	724.2	739.9	755.4	771.3	787.0	802.8
29.1	798.3	816.1	833.8	851.3	869.2	886.9	904.7
29.0	888.5	908.2	927.9	947.6	967.4	987.2	1007.0
28.9	979.0	1000.7	1022.4	1044.2	1065.9	1087.8	1109.6
28.8	1069.9	1093.5	1117.3	1141.1	1164.8	1188.8	1212.6
28.7	1161.1	1186.7	1212.5	1238.3	1264.1	1290.0	1315.9
28.6	1252.5	1280.3	1308.1	1335.9	1363.8	1391.6	1419.5
28.5	1344.3	1374.2	1404.0	1433.8	1463.7	1493.6	1523.5
28.4	1436.4	1468.4	1500.2	1532.1	1563.9	1595.9	1627.9
28.3	1528.5	1562.9	1596.8	1630.7	1664.5	1698.6	1732.7
28.2	1621.5	1657.7	1693.7	1729.6	1765.6	1801.7	1837.9
28.1	1714.6	1752.8	1790.9	1828.9	1967.0	1905.2	1943.4
28.0	1808.1	1848.3	1888.5	1928.6	1968.8	2009.0	2049.3
27.9	1901.9	1944.2	1986.4	2028.6	2071.0	2113.2	2155.6
27.8	1996.0	2040.4	2084.7	2128.9	2173.5	2217.8	2262.3
27.7	2090.5	2136.9	2183.4	2229.6	2276.3	2322.7	2369.3
27.6	2185.2	2233.8	2282.4	2330.7	2379.4	2428.0	2476.7
27.5	2280.3	2331.1	2381.7	2432.2	2482.9	2533.6	2584.5
27.4	2375.8	2428.7	2481.4	2534.1	2586.8	2639.6	2692.7
27.3	2471.6	2526.7	2581.3	2636.2	2691.1	2746.0	2801.3
27.2	2567.8	2625.0	2681.9	2738.9	2795.9	2852.9	2910.3
27.1	2664.3	2723.6	2782.6	2841.8	2901.0	2960.2	3019.7
27.0	2761.2	2822.6	2883.9	2945.1	3006.5	3067.9	3129.5

The following table will be of use in the reconnaissance, and in arranging the height of the signal and observing tripod to be erected for long lines over water or level land. The line of sight from the telescope to the signal should never be allowed to pass less than 6 feet above the ground at the tangent point.

Difference in feet between the apparent and true level at distances varying from 1 to 66 miles.

Distance, miles.	Difference in feet for—			Distance, miles.	Difference in feet for—			Distance, miles.	Difference in feet for—		
	Curvature.	Refraction.	Curvature and refraction.		Curvature.	Refraction.	Curvature and refraction.		Curvature.	Refraction.	Curvature and refraction.
1	0.7	0.1	0.6	23	353.0	49.4	303.6	45	1351.2	189.2	1162.0
2	2.7	0.4	2.3	24	384.3	53.8	330.5	46	1411.9	197.7	1214.2
3	6.0	0.8	5.2	25	417.0	58.4	358.6	47	1474.0	206.3	1267.7
4	10.7	1.5	9.2	26	451.1	63.1	388.0	48	1537.3	215.2	1322.1
5	16.7	2.3	14.4	27	486.4	68.1	418.3	49	1602.0	224.3	1377.7
6	24.0	3.4	20.6	28	523.1	73.2	449.9	50	1668.1	233.5	1434.6
7	32.7	4.6	28.1	29	561.2	78.6	482.6	51	1735.5	243.0	1492.5
8	42.7	6.0	36.7	30	600.5	84.1	516.4	52	1804.2	252.6	1551.6
9	54.0	7.6	46.4	31	641.2	89.8	551.4	53	1874.3	262.4	1611.9
10	66.7	9.3	57.4	32	683.3	95.7	587.6	54	1945.7	272.4	1673.3
11	80.7	11.3	69.4	33	726.6	101.7	624.9	55	2018.4	282.6	1735.8
12	96.1	13.4	82.7	34	771.3	108.0	663.3	56	2092.5	292.9	1799.6
13	112.8	15.8	97.0	35	817.4	114.4	703.0	57	2167.9	303.5	1864.4
14	130.8	18.3	112.5	36	864.8	121.1	743.7	58	2244.6	314.2	1930.4
15	150.1	21.0	129.1	37	913.5	127.9	785.6	59	2322.7	325.2	1997.5
16	170.8	23.9	146.9	38	963.5	134.9	828.6	60	2402.1	336.3	2065.8
17	192.8	27.0	165.8	39	1014.9	142.1	872.8	61	2482.8	347.6	2135.2
18	216.2	30.3	185.9	40	1067.6	149.5	918.1	62	2564.9	359.1	2205.8
19	240.9	33.7	207.2	41	1121.7	157.0	964.7	63	2648.3	370.8	2277.5
20	266.9	37.4	229.5	42	1177.0	164.8	1012.2	64	2733.0	382.6	2350.4
21	294.3	41.2	253.1	43	1233.7	172.7	1061.0	65	2819.1	394.7	2424.4
22	322.9	45.2	277.7	44	1291.8	180.8	1111.0	66	2906.5	406.9	2499.6

$$\text{Curvature} = \frac{\text{square of distance}}{\text{mean diameter of earth}}$$

$$\log \text{curvature} = \log \text{square of distance in feet} - 7.6209807.$$

$\text{Refraction} = \frac{K^2}{R} m$, where K represents the distance in feet, R the mean radius of the earth ($\log R = 7.3199507$), and m the co-efficient of refraction, assumed at .070, its mean value, sea-coast and interior.

$$\text{Curvature and refraction} = (1\text{–}2\ m)\ \frac{K^2}{2R}.$$

Or, calling h the height in feet, and K the distance in statute miles, at which a line from the height h touches the horizon, taking into account terrestrial refraction, assumed to be of the same value as in the above table (.070), we have—

$$K = \frac{\sqrt{h}}{.7575} \qquad h = \frac{K^2}{1.7426}.$$

The following examples will serve to illustrate the use of the preceding table:

I.—*Elevation of instrument required to overcome curvature and refraction.*

Let us suppose that a line, A to B, was 18 miles in length over a plain, and that the instrument could be elevated at either station, by means of a portable tripod, to a height of 20 or 30 or 50 feet. If we determine upon 36.7 feet at A, the tangent would strike the curve at the distance represented by that height in the table, viz, 8 miles, leaving the curvature (decreased by the ordinary refraction) of 10 miles to be overcome. Opposite to 10 miles we find 57.4 feet, and a signal at that height erected at B would, under favorable refraction, be just visible from the top of the tripod at A, or be on the same apparent level. If we now add 8 feet to tripod and 8 feet to signal-pole, the visual ray would certainly pass 6 feet above the tangent point, and 20 feet of the pole would be visible from A.

II.—*Elevations required at given distances.*

If it is desired to ascertain whether two points in the reconnaissance, estimated to be 44 miles apart, would be visible one from the other, the natural elevations must be at least 278 feet above

mean tide, or one 230 feet, and the other 331 feet, &c. This supposes that the intervening country is low, and that the ground at the tangent point is not above the mean surface of the sphere. If the height of the ground at this point should be 200 feet above mean tide, then the natural elevations should be 478, or 430 and 531 feet, &c., in height, and the line barely possible. To insure success, the theodolite must be elevated, and at both stations, to avoid high signals.

III.—*To determine whether the line of sight between two stations would pass above or below the summit of an intervening hill, and how much in either case.*

h_1 = height of lower station. d_1 = distance h_1 to h_2.
h_3 = height of higher station. d_2 = distance h_2 to h_3.
h_2 = height of intervening hill.

Example I.

h_1 = 600 feet.	600 feet strikes horizon at	32.3 miles,
h_3 = 2000 feet.	64 − 32.3 = 31 7 miles	= 577 feet,
h_2 = 1340 feet.	31.7 − 10 = 21.7 miles	= 270 feet,
d_1 = 54 miles.	2000 − 577 feet	= 1423 feet,
d_2 = 10 miles.	$\frac{64}{10} = 6.4$ and $\frac{1423}{6.4}$	= 222.3 feet,

and h_1, or height of line at h_2 = 1423 + 270 − 222.3 = 1470 feet.

Hence, the line passes 130.7 feet above the intervening hill and the stations are intervisible.

Example II.

h_1 = 900 feet.	900 feet strikes horizon at	39.4 miles,
h_3 = 3600 feet.	80 miles − 39.4 miles = 40.6 miles	946.0 feet,
h_2 = 1980 feet.	40.6 − 25.0 = 15.6 miles	139.8 feet,
d_1 = 55 miles.	3600 feet − 946 feet	2654.0 feet,
d_2 = 25 miles.	$\frac{80}{25} = 3.2$ and $\frac{2654}{3.2}$	829.4 feet,

and h = 2654 + 139.8 − 829.4 = 1964.4 feet.

Hence, the summit at h_2 is 15.6 feet higher than the line of sight, and the two stations are not intervisible.

If we elevate the instrument 60 feet at h_3, the line would pass clear of h_2, or its height at that point would be 2,006 feet.

The question of intervisibility may be also determined by the following formula, in which the co-efficient of refraction is reduced to .065.

$$h = h_1 + (h_3 - h_1)\frac{d_1}{d_1 + d_2} - 0.5803\, d_1\, d_2.$$

Example III.

For which we employ the same data as in Example I.

					log's.
$(h_3 - h_1)$ =	1400 feet	log $(h_3 - h_1)$	= 3.14613	d_1 = 54 miles	1.73239
d_1 =	54 miles	log d_1	= 1.73239	d_2 = 10 miles	1.00000
$d_1 + d_2$ =	64 miles	Co. log $(d_1 + d_2)$	= 8.19382	Constant	9.76365
	1181.2 feet		3.07234	313.4 feet	2.49604

and, hence, h = 600 feet + 1181.2 − 313.4 = 1467.8 feet.

NAMES OF STATIONS.

The station should be called after the popular designation of the hill or site on which it is situated; or after some peculiarity in the ground or formation well known in the neighborhood; or from the name of the owner or tenant of the land; or where neither owner nor distinctive traits are to be found, by such designation as, in the opinion of the assistant, will best serve to attract

attention to the special locality. The same name should not occur twice within the same section of country. If it should be necessary to re-occupy the station and the center cannot be positively identified, the fact should be stated, and the approximate position be designated as No. 2, or by the year of the re-occupation, as, for instance, Red Hill[2] or Red Hill[86]. If the new position is merely in the vicinity of the old station, an entirely new name should be given to it.

SIGNALS.

The following signals are employed in the triangulation, according to circumstances:

I. The heliotrope carefully adjusted, on a stand or tripod, to the center of station, and the pointing watched and attended to by an employé trained for the duty. This signal is used when long lines are to be observed, or those rendered difficult by haze, smoke, or other impurities in the atmosphere.

II. A reflector, such as the frustum of a tin cone, mounted on a stout pole, supported and retained in position by a tripod, and by wire guys, if advisable.

III. And in short lines, the simple pole supported by a tripod.

The height of the pole and dimensions of the tripod; the boarding of the upper and lower part, or the color of the pole, whether black or white, or with alternate bands of each, to insure prompt recognition; and the character of the mark, if any, centered on the top, to be used for identification, will vary with and depend upon the length of the line; the altitude of the station; the background of the signal as seen from the points of observation, and the atmospheric difficulties to be overcome.

In regard to the diameter of the pole to be observed upon, it is evident that, in short lines, it should not exceed the size just sufficient to admit of its being distinctly seen, as all beyond that is a source of error, arising from the additional range given to the bisecting thread.

The diameter which subtends an angle of one second at one mile is 0.307 of an inch, and, hence, at

20 miles it is 6.1 inches.	60 miles it is 18.4 inches.
40 miles it is12.3 inches.	80 miles it is 24.6 inches.

These proportions show that, for lines exceeding 15 miles in length, the diameter of the signal should not exceed one second in value; while, on the other hand, as the pole, when distinct, should be observed upon in preference to the heliotrope, and can be best seen during cloudy periods, it would be advisable, in long lines, to give to the pole nearly its full size; and it may be added that the height in signal adds to its visibility and distinctness.

A sketch of a tripod-signal is presented, the height of which varies from 15 to 65 feet. The solid part of the pole above the apex of the tripod should not exceed 8 or 10 inches in diameter, otherwise it would be too heavy to raise. When once in position, the upper part can be increased to the desired diameter by nailing on slats of light seasoned wood, or this addition may be partially made before raising.

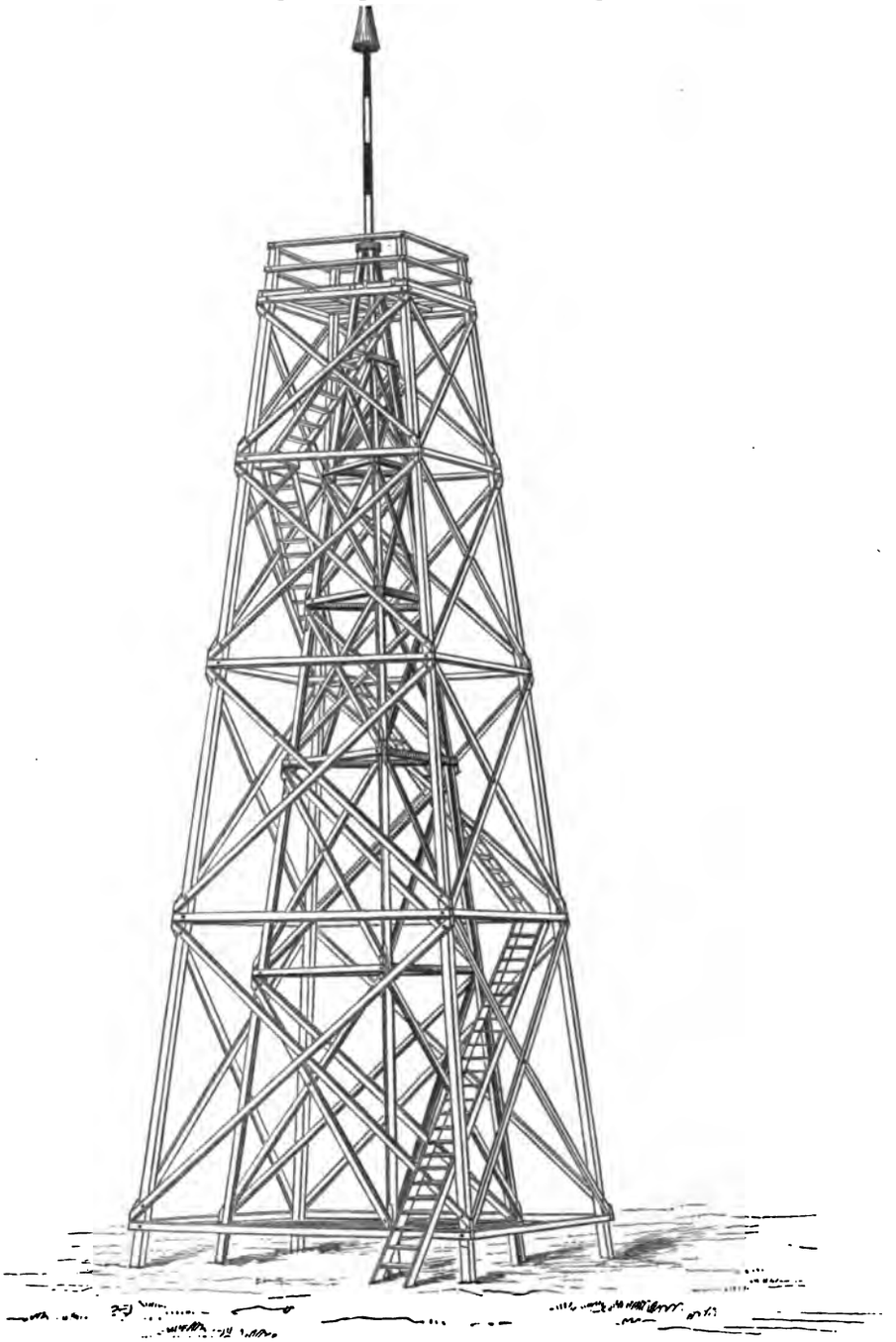
In the erection or use of any kind of signal no chance for an error in the determination of the angle should be allowed by a want of care and precision in centering the mark to be observed upon, exactly over the center of the station. And should this source of error be at any time suspected, an examination is promptly made, and if found to exist, the distance and direction of the point, or center of the object actually observed upon from the center of the true station, should be measured and recorded for the correction of the angles. (See formula for eccentricity of signal.)

In no case should the foot of the pole be inserted in or be allowed to come in contact with the earthenware cone or other article buried as an underground mark. Six or eight inches of earth, carefully packed above the cone or block, and upon this a square foot or so of board, upon which

the pole can rest, will be sufficient to afford a foothold when such is necessary, and, at the same time, to prevent any displacement of or injury to the mark in case the pole should be roughly handled or blown down.

OBSERVING TRIPODS AND SCAFFOLDS.

The tripod and scaffold, which are frequently erected for the elevation of the instrument and observer, in order to obtain a longer length of line or to escape the troubled condition of the at-



mosphere usually lying immediately over the low, flat lands bordering the coasts and shores, vary in height from 10 to 60 feet, and are made of scantling purchased for the purpose or of materials

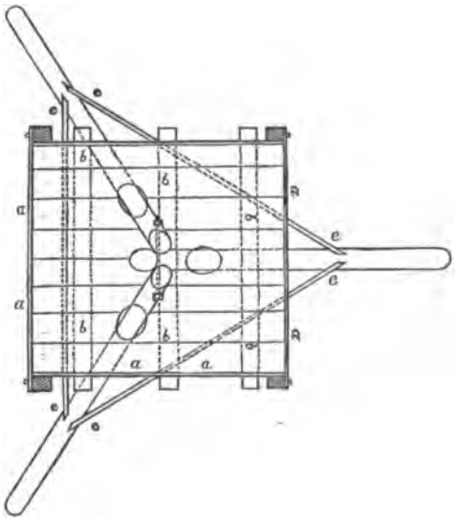
obtained from the forest, and are built in general accordance with the plans and specifications given in the annual reports. While a strict adherence to uniformity in the details of the construction is not possible, so much depending upon the means and facilities at the disposal of the assistant and upon a proper regard to economy, the general principles of strength and solidity, in both tripod and scaffold, are strictly observed by a proper spread and anchoring of the feet, a thorough bracing of the legs, and a compact fitting of the cap to the top of the tripod. A careful attention to these points will secure perfect immobility while the observations are being made and sufficient firmness to keep the scaffold entirely and always free from contact with the tripod, and to enable both structures to resist the most violent storms.

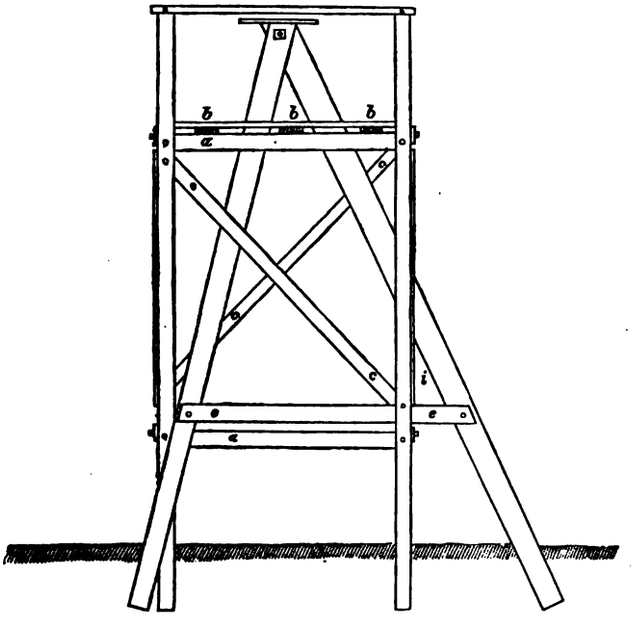
A drawing of such a tripod and scaffold is here given. It will be noticed that the diagonal braces of the tripod are spiked to the legs, one on the outside and the other on the inside, and that a space is left between the two braces where they cross. By drawing together these diagonals on each face of the tripod, the structure can be screwed up or stiffened, in case it becomes loose from shrinkage or other cause.

The following description and plan of a small portable tripod and scaffold may be of service, the height being well adapted for short lines on the southern coast:

Tripod.—This is made of three legs, 18 feet long and 6 inches in diameter, bolted together, 3 inches from the top, with an inch bolt, 16 inches long. After they are fitted together at the head, the spread being 13 feet, and before raising, one of the braces, *e e*, is screwed to the two outside legs, so as to keep them in place while raising the tripod by the third leg. It is then settled 2 feet in the ground, care being taken to level it by the braces *e e*, which are to be screwed on with wooden screws.

Scaffold.—Four posts, 16½ feet long and 5 inches square; 8 cross-pieces, 7 feet long and 6 inches by 1¼, *a a;* 3 cross-pieces, 7 feet long and 6 inches by 1, *b b;* 4 braces, 10 feet long and 5 inches by 1¼, *c c;* 8 flooring pieces, 7 feet 3 inches long and 9¼ by 1, with holes for the tripod-legs.





The end posts are those on which the braces and cross-pieces screw on the outside, and which are to be fastened together in pairs, when on the ground, so that it may be raised after the manner of a bedstead or house frame. The braces *c c* are to be bolted to the upper ends to steady the posts when raising them. All the holes in the posts, cross-pieces, and braces are to be identical as to plan and size, as also the pieces themselves, so as to have no mistakes, and when raised the scaffold fits to the tripod, as shown in the drawing. The scaffold is leveled by the cross-pieces and adjusted, and in firm position after the floor is on, so as to be free from the tripod. The floor is to

be 2 feet 10 inches below the top of the tripod. Three iron knees are screwed to the tripod-legs near the top, so that the triangular piece for the theodolite can be bolted to them. This piece is made of two pieces of one-inch plank, screwed together across the grain, and then painted. Holes are made in the floor for the tent-posts, and wire guys are sometimes required for the scaffold.

The above takes about two hours to put up. One large and two small wrenches are necessary; also a bag to contain the screws, nuts, &c. Should more than one be needed, they should be painted different colors, but be, in all other respects, exactly alike, in order that one can be used to repair the other.

THE UNDERGROUND STATION-MARKS.

The station-marks include the underground and surface marks; the former to be buried, and the latter to be thrown up for the preservation of the center and of the position of the station.

The requisites for an underground mark or one buried below the frost and plow line, and beyond the reach of ordinary accident or interference, say three feet, in the clear, below the surface, are: indestructibility, peculiarity, capacity to resist displacement in case it should be accidentally struck, cheapness, and, finally, want of value for any of the ordinary purposes of life, as a protection against cupidity. The following marks, partaking more or less of these essential qualities, have been adopted in the survey.

1. The frustum of a hollow stoneware cone, called the Hassler cone. The dimensions for primary stations are for the upper and lower diameters and the height 8, 12, and 15 inches, respectively.

2. One similar in shape to the preceding, but made of iron, and occasionally with a rim like that of a hat, encircling the larger diameter, upon which are inscribed in the casting the words "U. S. Coast Survey."

3. A hollow stoneware pyramid.

4. A short column of marble, granite, or sandstone, manufactured for the purpose, and in some cases placed above the cone, the top reaching within six inches of the surface.

5. A block composed of brick or stones and hydraulic cement.

6. A bottle with three others just below the surface pointing to the lower one.

The center of the station in the cone is either the center of the periphery or the intersection of two lines drawn on the head of a copper tack driven in a stub placed and packed inside of cone, and sometimes extending within a foot of the surface; or, when a block is used, by the intersection of cross-lines on the head of a copper bolt inserted for the purpose. The initials U. S. C. S. are occasionally cut upon the block.

Of these and other varieties of underground marks which have been adopted at times from choice and again from necessity, the stoneware or iron cones are clearly to be preferred.

THE SURFACE STATION-MARKS.

The surface marks are so varied in their character, and depend so much upon the nature of the ground, that no special rules can be established for their selection. What would be highly appropriate in one case would be equally inappropriate in another; and at many localities, such as those affected by the winds and waves and unavoidable as stations from the necessities of the work, no marks whatever can be arranged or erected with any hope of permanency, except at considerable expense. As the object of the surface marks is to secure the position and recognition of the station at any time hereafter, it is evident that the general principles which should govern in each case, to the full extent to which the locality will admit of their application, are, permanency or durability, facility with which they can be recognized, and the absence of value for any domestic or farm purpose. The following methods of marking have been employed in the survey:

1. A hole, an inch or two in diameter, drilled in the rock on which the signal stands, and filled with lead, sulphur, or a copper bolt.

2. In the case of earth, a stone block, pillar, or post, marking the center of the station, and three others, two in line and one at right angles, equidistant from the center.

3. A large rock rolled to center and there sunk, or a block constructed of brick or stones and hydraulic cement, with the usual drilled hole and bolt of lead, copper, or sulphur.

4. An iron screw-pile, the center being marked on the cap, and also on a stub inside of tube, the top of stub having been previously covered with a disc of copper sheeting, on which the initials of the survey and date are punctured.

5. Owing to the requirements of the survey, many of the stations are located on the immediate line of coast, or on the margin of the low shores of the Gulf and Southern sounds. When situated on a sand knoll or hill, the point is secured by a screw-pile, or a stone block packed in a box or framework of wood; and by the introduction, at and around the station point, of clay or marsh mud, or other foreign substance convenient to the locality, and the knoll protected from the moving effect of the wind by abattis, constructed of a circle or circles of stakes intertwined with brushwood, an accumulation of sand being less objectionable than the denuding of the station-marks.

6. Or, in case of a raised beach, subject to being washed away during a heavy gale, or by the slower action of the currents, beside the usual station-marks, a point of reference is established back from the shore-line, and its relation to the center of the station, determined by a careful measurement of the distance and azimuth between the two, so that the duplicate point may be used, or another established as an eccentric station, should such be needed at any time hereafter.

7. When the station is situated on the margin of a marsh, or a wooded swamp, it is secured by a screw-pile, or long pieces of scantling forced as far as possible into the yielding soil, and projecting two or three feet above the surface.

8. Beside the natural elevations occupied as stations, those of an artificial character have been made available whenever it was expedient to do so, such as light-houses, towers, steeples, houses, barns, &c. The center of the station, when it can be marked, is designated, in the case of stone or cement, by the usual drilled hole and bolt; or, where metal is found, by a point within a triangle, both deeply engraved; or, in the case of wood, by a wooden pin driven in an auger-hole bored within a triangle cut, or formed by copper tacks, or by a piece of copper sheeting nailed to the wood and marked as above.

9. In addition to one or the other of the above marks, to be selected according to its special applicability to the case, the position is secured, whenever possible, by a circular trench, either left open or filled with charcoal or other substance foreign to the position, and then covered up, or by a mound of earth or pyramid of stones covering the surface-marks.

10. By points of reference, such as measurements and magnetic courses, from the center of the station to rocks in situ, stone walls, houses, trees, stakes in the prolongation of the lines to other stations, or to some prominent hill or building, and to other more or less permanent objects, artificial or natural. When the points referred to are within measuring distance, they should be designated by a triangle or other appropriate mark.

11. By a written description and topographical plan of the ground, its surroundings, and approaches, including the station-marks, the points of reference, and the courses and distances thereto. The name of the owner or tenant of the land, and of the resident or neighbor who has been requested to take charge of the station, or of others who will know most about its position, should be given.

12. And, finally, at important stations, by views and sketches of the locality and its peculiarities, from different points, as a further means of identification.

The stone pillars or posts are from 4 to 6 inches square, and vary in length from 24 to 30 inches; the blocks or monuments from 8 to 24 inches square, and from 18 to 20 in depth, and in all cases sunk nearly level with the ground. The usual cross-lines to define the actual center are drawn on these as well as on the bolts, and the letters U. S. C. S. in some cases cut upon the stone.

The distance of the three surface-marks, whether of stone or cedar, as referred to in No. 2, should be uniformly 6 feet from the center of station, unless a different distance is unavoidable from the nature of the ground. Each should have an arrow-head on it, pointing to the center of the station, and they should be placed north, south, and east, in order to facilitate the search, should one or two be covered up or lost. The distances and courses, however, are always given in the description of stations.

Too great care cannot be taken for the security and identification of the station.

THE OBSERVATIONS AND RECORDS.

Uniformity in conducting the field operations, and in the order in which the observations are recorded, should be strictly observed.

1. The number of the theodolite, its size, and the order of its graduation, whether from left to right, or right to left, should be entered at the commencement of each volume.

2. The angles should be measured in the direction of the increasing numbers on the circle, so that, in all cases, the first reading may be subtracted from the second, and the station first pointed at should be the first named.

3. Whatever may be the number of observations of which each series may consist, those taken in the direct position of the telescope should be separated from those taken in the reversed position.

4. If the telescope cannot be reversed, except by being lifted from its supports, the verniers should be re-read after reversal.

5. In the determination of important angles, the pointings should be so arranged as to fall upon as many different parts of the circle as the proposed number of measurements will permit, with a view to eliminate any error of graduation or eccentricity.

6. When the angle measured is subject to a correction for phase, the direction of the sun and the time should be given, as well as the form and dimensions of the tin cone or other reflecting object observed upon.

7. The closing of the circle in the secondary triangulation, or the measurement of the angle between the last station and the first, should be made with the same care with which the regular angles were determined.

8. The observations should follow in the record in the order in which they were taken. The angle between the signal and sun for computing the correction for phase should follow the observations to which it belongs; and when the station is eccentric, the fact is mentioned at the top of the page, and the additional angle and distance required recorded in full.

Economy of pages or space in the record-book is of little consequence in comparison with irregularities in the record.

It would add to the completeness of the season's records if a plan of the triangulation executed should be attached to the first volume; and, also, if at the commencement of each station the telescope, fixed at zero, should be pointed at the first station, and then, following the graduation, to the next, and so on to the starting point, completing the horizon and recording the reading in each case, so as to obtain the approximate value of the different angles to be measured.

The following abbreviations denoting the kind of signal observed upon, its appearance, &c., are those generally adopted in the survey. Too great detail in this matter is not considered necessary, having no weight in the computations; as in cases where the angle is really believed to be affected by the condition of the atmosphere in either magnifying, distorting, or giving too great motion to the signal, the observer will say so, and, after giving his reasons, will reject the observations; and if he does not, the computer is rarely, if ever, authorized to do so.

ABBREVIATIONS.

SIGNAL.

Heliotrope	H.
Cone	C.
Pole	P.
Tuft	T.
Crotch	Cr.

DEGREE OF VISIBILITY.

Distinct	dt.
Bright	br.
Faint	ft.
Flaming	fl.
Diffuse	dif.

STEADINESS.

Steady	st.
Tremulous	tr.
Moving	mg.

WEATHER.

Clear	cl.
Flying clouds	fy. cl.
Cloudy	cly.

FIGURE.

Round	r.
Oval	ov.
Irregular	ir.

SIZE.

Point	pt.
Small	sm.
Large	lg.

Two classes of theodolites are used in the triangulation, one termed direction and the other repeating instruments, the first being devoted exclusively to the primary work. The record-books are prepared with printed headings for the different columns, showing the order in which the details are to be entered.

Form for direction instruments.

Station, Blue Hill, Mass.—Position IV.—Date, September 25, 1845.—Observer, A. D. B.—Recorder, J. S.

Series and number.	Object observed.	Time.	Tel.	Mic.	°	′	d	d	Mean.	Correction for run.	Corrected mean.	Remarks.
		h. m.										
II	Azimuth mark.	8 43	D	A	271	31	49.0	50.2				Position 0° upon 180°.
21	d. st.			B			55.0	55.0				Levels { A 73d. B 99.
				C			60.2	62.0				
							54.7	55.7	55.2	—.1	55.1	Temp. 59° F.
22	Manomet H.	47	D	A	35	26	48.4	49.2				Atmospheric pressure....
	m. l.; m. ro.; st.			B			50.5	51.8				Weather foggy, ci.-cu.
				C			56.0	58.0	52.3	—.1	52.2	Wind N. E., light.
23	Manomet H.	50	R	A	215	26	37.0	40.0				
	As before.			B			39.0	41.8				
				C			42.0	46.8	42.1	—.2	41.9	
24	Azimuth mark.	52	R	A	91	31	44.0	44.0				
	d. m. st.			B			43.0	45.0				
				C			49.0	51.0	46.0	—.2	45.8	
25	Thompson H.	9 02	R	A	124	50	38.0	39.0				
	sm.; ro.; st.			B			40.8	41.0				
				C			50.0	51.0	43.3	—.5	42.8	
26	Etc., etc.											N. B.—Microscopes adjusted for position and run, September 16, 1845. 300 div. of micr. = 301″.2 = mean of three microscopes.

The progress of the work is frequently advanced by the use of a reference-mark, to which all the other directions are referred. This mark is usually the meridian or azimuth mark used in the determination of the astronomical azimuth, or one of the regular signals likely to be the most constantly distinct, or one erected for the purpose, the last being rarely necessary.

No special form is required for the observations to determine the errors of run of the reading-microscopes. They are entered in the preceding form of record as they are made, generally on first occupying a station, and principally for the purpose of verifying the imposed condition that the adjustment of the microscopes to the graduation of the instrument should be so far perfected that five revolutions of the micrometer-screw do not overrun or underrun a five-minute space on the circle by a greater error than two seconds. The regular observations or readings on all parts of the circle supply the data for determining the mean error of runs used in the correction of those observations.

CORRECTION FOR ERROR OF RUNS.

a = first reading. Turn the micrometer-screw in the direction of the increasing numbers on its head until the cross-wire intersects the nearest five-minute division on the circle, and read the number of turns and parts of a revolution.

b = second reading. Reverse the movement of the screw and continue the backward motion of the cross-wire until it reaches the nearest five-minute division, and read as before.

Let the following data represent the mean of a number of observations taken from the record,

the degrees and minutes read off from the circle being 65° 20′. As the micrometer over-runs, the correction is subtractive from the mean of the two readings.

$a = 65\ 20 + 4\ 42.6$ $\qquad r = a - b = +\ 0\ 1.8$

$b = 65\ 20 + 4\ 40.8$ $\qquad m = \frac{a+b}{2} = 4\ 41.7$

Correction to $a = \frac{r}{300''} a$ ..... $= -\ 1.70$

Correction to $b = \frac{r}{300''} (b - 300'')$ ..... $= +\ 0.12$

Correction to $m = \frac{1}{2}\left(\frac{a + b - 300''}{300''}\right) r$ ..... $= -\ 0.79$

and, hence, the corrected mean of the two readings = 65° 24′ 40″.9.

Tables of double entry are constructed, by means of which the corrections for run can be taken out by inspection. The arguments are the number of minutes and seconds in the observation and the value of $\pm r$.

Form for repeating-instruments.

[Mount Rafinesque, August 21, 1875. Observer, ——; recorder, ——.]

Stations.	Time.	D R	Repet'ns.	A	B	C	D	Mean of verniers.	Angles.	Mean of D and R.	Remarks.
	h. m.			° ′ ″	′ ″	′ ″		″	° ′ ″	° ′ ″	
Mount Equinox.	7 05 a. m.			70 04 13	04 35	04 32		26.6			
Greylock.									70 26 17.5		
		D	3	281 23 10	23 35	23 12		19.0		70 26 15.8	
									14.0		
Do.		R	3	132 41 52.5	42 21	41 58		03.8			
				132 41 52.5	42 21	41 58		03.8			
									70 26 17.6		
Do.		R	3	344 00 45	01 05	01 00		56.7		70 26 16.8	
									16.1		
		D	3	195 19 36	19 53	19 46		45.0			
Greylock.	7 30 a. m.			195 19 36	19 53	19 46		45.0			
Perry's Peak.									40 06 53.9		
		D	3	315 40 20	40 35	40 25		26.7		40 06 54.7	
									55.5		
Do.		R	3	76 01 05	01 20	01 15		13.3			
				76 01 05	01 20	01 15		13.3			
									40 06 55.2		
Do.		R	3	196 21 51	22 10	21 56		59.0		40 06 55.1	
									55.1		
		D	3	316 42 38	42 50	42 45		44.3			

NUMBER OF OBSERVATIONS.

The number of observations required for the determination of any one angle must depend upon the desired closeness of the result and the character of the instrument.

The direction-instrument requires that it should be turned on its stand, or changed in position, in order that the direction of any one line, and consequently of all, should fall upon different parts of the circle as the only security against errors of graduation. The number of positions varies from five to twenty-one, of nearly equal arcs; and in each position the circuit of the horizon is made, giving the direction of each line by two observations, one in the direct and the other in the reversed position of the telescope. These circuits, or series, are repeated in each position until two to five values of each direction are obtained. Each angle is, therefore, determined by from 35 to 63 measurements in the direct and a like number in the reversed position of the telescope.

When repeating theodolites are employed for the primary work, the number of measurements of each angle varies from 90 to 120, divided into sets of three or six repetitions, direct, and the same number, reversed. In the secondary triangulation, in which the length of the sides varies from five

to forty miles, and especially in cases where such triangulation is the most extended which the nature of the country will admit of, and the results of which become in consequence of primary importance, the angle should be determined by not less than six sets, each consisting of six repetitions in the direct and six in the reversed position of the telescope, or twelve sets of three in the D and three in the R. If the triangles belong to the tertiary series, the sides being five miles and under in length, the number of sets will vary from three to six, according to the distance along the coast over which the chain will extend before a verification can be effected by connection with a line of the secondary triangulation, or by the measurement of a subsidiary base. In this class of angles, each set is made up of three repetitions in the D and three in the R, in order to increase the number of separate results with a view to comparison and the elimination of error.

While a minimum number of measurements may be prescribed for each angle belonging to the orders of triangulation referred to, the maximum must be left entirely to the judgment of the observer. In forming this judgment, the known or supposed character of the instrument in regard to accuracy of graduation or the reverse; the elevation of the instrument; the differently refracting media through which the line of sight passes, as over alternate sections of land, marsh, and water; the lateral refraction incident to long lanes opened through the forest or swamp; the appearance of the signal and the condition of the atmosphere, will be taken into consideration, and the observer, from one cause or another, may deem it advisable to multiply his observations until he becomes satisfied that he has obtained the value of the angle, or until an apparently discordant result is neutralized or proved to be exceptional.

In the case of auxiliary points determined with a concluded angle, such as light-houses, spires, chimneys, &c., and others specially thrown in between the secondary and tertiary stations to facilitate the operations of the topographical and hydrographical parties, a single measurement of six repetitions, three in the D and three in the R, will be sufficient. Care should be taken, however, that the unobserved angle should not greatly differ from 90°, and especially that each determination of this character should have its check or verification by a second determination from a different base.

LIMIT OF ERROR.

Assuming 3″ to be the limit of error of closing in triangles of the primary order, 6″ in those of the secondary, and about 12″ in those of the tertiary series, it is certain that to secure the desired degree of accuracy the observer should give his careful attention, seriatim, to the following points:

1. To the diameter and centering of each of the signals to be observed upon.
2. To the stability of the stand or tripod on which the theodolite rests.
3. To the centering of the instrument, and to its adjustment for parallax, collimation, and level.
4. To his personal comfort when observing, by having the eye and telescope at the same height when standing in a natural position, and by avoiding the necessity for any strain on the body or twist in the neck to look at any particular signal.
5. To the preservation of the levels while observing, and to a certain degree of rapidity in the pointing consistent with a clear and decided bisection of the signal.
6. To taking the different sets required for the determination of the angle, part on one day and part on another, or dividing them between the a. m. and p. m. periods for observing.
7. To declining to observe under any manifestly improper or doubtful condition of the atmosphere, as shown principally by the signals.

The observations taken on a day when the sky is entirely overcast from sunrise to sunset are the most reliable, and next in value are those taken during a calm afternoon when the sky is wholly clear. The a. m. observations made before the sun has dissipated the vapors of the night, or has quieted down the irregularities in the lower stratum of the atmosphere by an equalization of the temperature, are believed to be the most uncertain. These irregularities may be recognized by an unusual refraction or elevation of the signals above the horizon, beside the jumping, duplication, or distortion of the image, and the greater the vertical refraction, the greater will be the probability of a slight lateral deviation in the ray.

After the preceding precautions have been strictly observed and the prescribed number of

measurements taken, the next step will be to examine the separate values of each angle with a view to ascertain the probable accuracy of the pointings.

The probable error of an angle in the primary triangulation should not exceed 0″.3, and in the secondary 0″.7.

It is not improbable that after the prescribed number of sets has been taken a critical examination of the values, and of the circumstances under which they were obtained, will show that additional measurements are necessary or advisable. There are other errors beside those of pointing, which, notwithstanding every precaution, may largely increase the probable error of the resulting angle. The fact of an agreement among separate measurements is not a proof of having obtained the correct value of the angle, unless those measurements were made under different conditions of the atmosphere, and during a. m. and p. m., to guard against an altogether one-sided refraction and illumination of the signal.

PROBABLE ERROR.

o, o^1, o^2, &c. = the observed values of the angle.
n = number of such values, or sets of repetitions.
x_0 = their arithmetical mean.
Δ = differences between x_0 and o, o^1, &c.
Σ = symbol denoting sum.
ε = mean error.
r = probable error of any one of the observed values.
r_0 = probable error of x_0.

$$\varepsilon=\sqrt{\frac{\Sigma\Delta^2}{n-1}} \qquad r=0.6745\sqrt{\frac{\Sigma\Delta^2}{n-1}} \qquad r_0=\frac{r}{\sqrt{n}}$$

o, o^1, o^2, &c.	x_0	Δ	Δ^2
° ′ ″	″	″	
45 44 22.5	23.5	−1.0	1.00
24.8		+1.3	1.69
21.0		−2.5	6.25
21.4		−2.1	4.41
26.6		+4.1	16.81
22.3		−1.2	1.44
25.5		+2.0	4.00
24.6		+1.1	1.21
21.0		−2.5	6.25
23.3		−0.2	0.04
25.5		+2.0	4.00
23.5		0.0	0.00
45 44 23.5		20.0	47.10

$n-1=11$ co. log 11 ..... 8.95861
$\Sigma\Delta^2=47.10$ log 47.10 ..... 1.67302
0.63163
log ε ..... 0.31582 $\varepsilon=$ 2″.07
Const. .6745 log const 9.82998
log r ..... 0.14580 $r=\pm$1″.40
log $\sqrt{12}$ 0.53959
log r_0 ..... 9.60621 $r_0=\pm$0″.40

Or, using only the differences or residuals, according to the formulæ given by Schott, page 308 Coast Survey Report of 1856, and without employing logarithms,

$$r=0.845347\frac{\Sigma\Delta}{\sqrt{n(n-1)}}, \text{ or } r=0.84\frac{\Sigma\Delta}{n-1} \qquad r=0.845\frac{20}{11.49}=1''.47$$

$$r_0=\frac{r}{\sqrt{n}} \qquad r_0=\frac{1.47}{\sqrt{12}}=0''.42$$

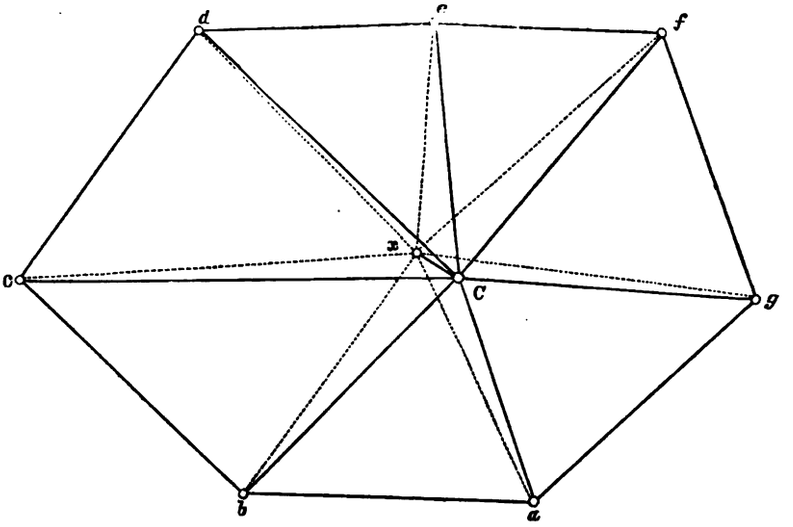
The number of measurements required for each angle having been taken, the next step will be to apply to each, or to their arithmetical mean, as the case may be, the corrections for phase, or eccentricity of the object observed upon, or those due to the occupation of an eccentric station. The angles in the abstract being corrected accordingly, if any such correction should be necessary,

the final computation of the triangle sides will be commenced by computing the spherical excess, and by the distribution among the angles of the error found in the triangle.

The examples given in illustration of the following formulæ for reduction to center of station, phase in tin cone, eccentricity of signal, and spherical excess, are believed to cover every possible case.

REDUCTION TO CENTER OF STATION.

When the center of the station cannot be occupied, the theodolite is adjusted at a point as close as possible to the center, which new point is called the eccentric station. To be able to reduce the angles measured at the eccentric to the center of the true station, it is necessary that the distance between the two should be measured with the utmost exactness, and that the angle at x, between the true center and one of the stations of the triangle, be carefully determined.



C = center of station.
x = eccentric position of instrument.
r = the distance Cx.
o = the angle at x between two signals, a and b.
y' = the angle at x between C and the left-hand signal a.
a = the distance Ca.
b = the distance Cb. log sin 1″ = 4.6855749
C = the unknown angle at C. co. log sin 1″ = 5.3144251

The signals are all supposed to be situated to the right of C, following each other in azimuthal order.

$$C = o + \frac{r \sin (o + y')}{b \sin 1''} - \frac{r \sin y'}{a \sin 1''}$$

The sign given for each term of the formula will be governed by that of the sine of $o + y$ and of y'.

Example.—Triangle *axb*.

$r = 2^m.2145$ ..... log r.	0.3452757	$r = 2^m.2145$ ..... log r.	0.3452757
$o + y' = 89° 45' 20''$; log sin $(0+y')$	9.9999960+	$y' = 28° 29' 30''$; log sin y'.	9.6785465+
co. log b	6.0855685	co. log a	6.1870866
co. log sin 1″.	5.3144251	co. log sin 1″	5.3144251
	1.7455653..	+55″.66	1.5253339... −33″.52
		−33″.52	

C = 61° 15′ 50″ + 22″.14 = 61° 16′ 12″.14

Example.—Triangle *exg.*

$r = 2^m.2145$ ..... log r.	0.3452757	$r = 2^m.2145$ ..... log r.	0.3452757
$o + y^5 = 330° 20' 40''$; log sin $(0+y')$	9.6944163—	$y^5 = 236° 24' 15''$ log y^5..	9.9206249—
co. log g	6.0910863	co. log e	6.1429692
co. log sin 1''	5.3144251	co. log sin 1''	5.3144251
	1.4452034..	—27''.87	1.7232949... +52''.88
		+52''.88	

$$C = 93° 56' 25'' + 25''.01 = 93° 56' 50''.01$$

When a large number of angles have been observed at an eccentric station, and different combinations are required, it is recommended that the directions to the different signals be arranged in the order of their azimuths, starting from the line *xc*, as shown in the following example, and that the correction for each line be computed by the formula

$$\frac{r \sin y^1}{a \sin 1''}$$

Angle.	Under 180°.	Over 180°.
$(o + y)$	+	−
y^1	−	+

By adding or subtracting the corrections for the two lines inclosing the special angle according to the signs given in this table, the reduction to the center and its sign will be obtained. The direction to the right-hand signal always represents $(o + y)$.

Example.

	x to C 0° or 180°	x to a y^1	x to b y^2	x to c y^3	x to d y^4	x to e y^5	x to f y^6	x to g y^7
Direction.	00° 00' 00''	28° 29' 30''	89° 45' 20''	139° 28' 40''	185° 32' 10''	236° 24' 15''	280° 14' 50''	330° 20' 40''
Log Sine		9. 67855	9. 99999	9. 81274	8. 98441	9. 92062	9. 99302	9. 69442
Log r		0. 34528	0. 34528	0. 34528	0. 34528	0. 34528	0. 34528	0. 34528
Co. log. dist.		6. 18709	6. 08587	5. 92010	5. 99965	6. 14297	6. 04576	6. 09109
Co. log sin 1''		5. 31443	5. 31443	5. 31443	5. 31443	5. 31443	5. 31443	5. 31443
		1. 52535	1. 74557	1. 39255	0. 64377	1. 72330	1. 69849	1. 44522
		"	"	"	"	"	"	"
Correction.		a=33. 52	b=55. 66	c=24. 69	d=4. 40	e=52. 88	f=49. 94	g=27. 88

Hence, the corrections would be, for the angle between

						"		"			° ' "	
a and b	=	+	55.66	−	33.52	and C =	61	15	50	+ 22.14 =	61	16 12.14
a " c	=	+	24.69	−	33.52	" C =	110	59	10	− 8.83 =	110	59 01.17
a " d	=	−	4.40	−	33.52	" C =	157	02	40	− 37.92 =	157	02 02.08
c " e	=	−	52.88	−	24.69	" C =	96	55	35	− 77.57 =	96	54 17.43
f " g	=	−	27.88	+	49.94	" C =	50	05	50	+ 22.06 =	50	06 12.06
g " a	=	+	33.52	+	27.88	" C =	58	08	50	+ 61.40 =	58	09 51.40
f and a	=	+	33.52	+	49.94	and C =	108	14	40	+ 83.46 =	108	16 03.46

The above eccentricity may be considered much greater than the average. Occasionally it is very small, as when, by some mistake, the instrument, mounted on a high tripod, cannot be adjusted exactly over the center of station, but a few inches off, in one direction or the other. The distance and direction, in all cases, should be most carefully determined; and, if the eccentricity is large, the triangle sides, or distances, should be re-computed with the observed angles corrected by an approximate reduction to the center.

CORRECTION FOR PHASE IN TIN CONES USED AS SIGNALS.

x = station of observer.
C = the sun; or, xC, the azimuth of sun.
y = angle between sun and reflecting cone.
a = distance between observer and cone.
r = mean radius of cone.

If the pointing is made on the bright reflecting line exhibited by the cone, then

$$\text{correction} = \pm \frac{r \cos \frac{1}{2}y}{a \sin 1''}$$

but if there is no such reflection, and the pointing be made on the white illuminated part of the cone,

$$\text{correction} = \pm \frac{r \cos^2 \frac{1}{2}y}{a \sin 1''}$$

Examples.

Bright phase.		White phase.	
$r=0^m.215 \log r$	9.33244	$r=0^m.215 \log r$	9.33244
$y=89° 40'$, log cos $\frac{1}{2}$ $89° 40'$.	9.85074	$y=89° 40'$, log cos $\frac{1}{2}$ $89° 40'$.	9.85074
$a=18206^m$, co. log 18206^m	5.73979	log cos $\frac{1}{2}$	9.85074
co. log sin $1''$	5.31443	$a=18206^m$, co. log 18206	5.73979
		co. log sin $1''$	5.31443
correction $=1''.72$	0.23740	correction $=1''.22$	0.08814
$r=0^m.215 \log r$	9.33244	$r=0^m.215$, log r	9.33244
$y=176°$, log cos $\frac{1}{2}$ $176°$	8.54282	$y=176°$, log cos $\frac{1}{2}$ $176°$	8.54282
$a=18206^m$, co. log 18206^m	5.73979	log cos $\frac{1}{2}$	8.54282
co. log sin $1''$	5.31443	$a=18206^m$, co. log 18206^m	5.73979
		co. log sin $1''$	5.31443
correction $=0''.08$	8.92948	correction $=0''.003$	7.47230

The bright phase belongs exclusively to the curved cones usually employed as signals in the primary and secondary triangulation.

The line of reflection, or the illumination, is always on the same side with the sun. If, therefore, the direction to the second signal is on the same side of the reflecting cone as the sun, the correction is additive to the observed angle; if on the opposite side, subtractive. If both signals are reflecting, or illuminated cones, the difference between the two additive or subtractive corrections, as the case may be, is the correction to be applied $\pm$ to the angle.

The angle between the sun and the signal should be measured immediately after completing the set of repetitions to which the correction must be applied. Should the angle be omitted, but the time be recorded, the azimuth of the sun may be computed. It should also be matter of record whether the cone reflects or merely shows white.

CORRECTION FOR ECCENTRICITY OF SIGNAL.

C=center of station.
x=the eccentric object, or part of signal observed upon.
r=the measured eccentricity.
a=the station of observer; also, the distance aC.

$$\text{Correction} = \pm \frac{r}{a \sin 1''}$$

Let us suppose, for example, that, during the occupation of the stations, a, b, c, and d, the pole at C was out of adjustment, or that x, the object observed upon, was not in the vertical of C, and that, to correct the error, the following measurements were made of the chord, or the perpendicular, from C to the different directions, viz: $0^m.155$, $0^m.293$, $0^m.182$, and $0^m.096$.

$r=0^m.155$, log r	9.19033	$r=0^m.293$, log r	9.46687
$a=6500^m$, log a sin 1″	8.49849	$b=8206^m$, log b sin 1″.....	8.59971
correction=4″.92	0.69185	correction=7″.36	0.86716
$r=0^m.182$, log r	9.26007	$r=0^m.096$, log r	8.98227
$c=12020^m$, log c sin 1″.....	8.76548	$d=1008^m$, log d sin 1″.....	8.68592
correction=3″.12	0.49459	correction=1″.98	0.29635

The diagram shows that in the triangles aCb, bCc, and cCd, the corrections to the observed angles should be

	″		″		″
For Cab.....	+4.92	Cbc.....	+7.36	Ccd.....	+3.12
For Cba.....	−7.36	Ccb.....	−3.12	Cdc.....	+1.98

SPHERICAL EXCESS.

Every angle measured in the ordinary operations of geodesy is a spherical angle, and consequently the sum of the three observed angles of any triangle should, theoretically, exceed 180°. This spherical excess, or ratio of the area of the triangle to the area of the sphere, becomes appreciable only when the sides are from four to five miles in length; and being so small a quantity in proportion to the errors of observation, it is entirely overlooked in triangles of the third order.

In the secondary and primary triangulation, the spherical excess is applied to determine the error due directly to the observations. One-third of the computed excess is deducted from each angle of the triangle, and the difference between the resulting sum of the angles and 180° is the error to be distributed.

Let a, b=triangle sides.

C=the included angle.

		log's.
A=equatorial radius.....	$6378206^m.4$,	6.8046985
B=polar radius	$6356583^m.8$,	6.8032238

e=the eccentricity=$\sqrt{1-\frac{B^2}{A^2}}$

e^2=0.00676815 ..... 7.8304700

L=mean latitude of the three stations.

$\epsilon=\frac{ab \sin C\,(1+e^2 \cos 2L)}{2A^2 \sin 1''}$, and making $m=\frac{1+e^2 \cos 2L}{2A^2 \sin 1''}$,

we have $\epsilon=ab \sin C m$.

The latitude being the only variable quantity in the formula m, we make the latter a constant by computing it for every 30′ of latitude likely to be embraced in the survey, and tabulate the results. The value of m for intermediate latitudes may be taken out by inspection, though such precision is only necessary in the very largest triangles.

Computation of m

L= 24°, 2L=48°, log cos 2L=9.8255109
e^2, log e^2 =7.8304700

log e^2 cos 2L=7.6559809
e^2 cos 2L=0.0045288
$1+e^2$ cos 2L=1.0045288
log $(1+e^2$ cos 2L)=0.0019624
log (2 A^2 sin 1″)=8.5960019

log *m* for lat. 24°=1.4059605

log sin 1″=4.6855749
log A^2=3.6093970
log 2=0.3010300

log (2 A^2 sin 1″)=8.5960019

Latitude.	log *m*.	Latitude.	log *m*.	Latitude.	log *m*.	Latitude.	log *m*.
24° 00′	1.40596	31° 30′	1.40533	39° 00′	1.40461	46° 30′	1.40385
24 30	592	32 00	528	39 30	456	47 00	380
25 00	588	32 30	524	40 00	451	47 30	375
25 30	584	33 00	519	40 30	446	48 00	369
26 00	580	33 30	514	41 00	441	48 30	364
26 30	576	34 00	509	41 30	436	49 00	359
27 00	572	34 30	505	42 00	431	49 30	354
27 30	568	35 00	500	42 30	426	50 00	349
28 00	564	35 30	495	43 00	420	50 30	344
28 30	559	36 00	491	43 30	415	51 00	339
29 00	555	36 30	486	44 00	410	51 30	334
29 30	551	37 00	481	44 30	405	52 00	329
30 00	547	37 30	476	45 00	400	52 30	324
30 30	542	38 00	471	45 30	395	53 00	319
31 00	1.40537	38 30	1.40466	46 00	1.40390	53 30	1.40314

Computation of spherical excess.

$a = 6500^m$, log a	3.81291	$a = 122755^m$, log a	5.08904
$b = 4615^m.6$, log b	3.66423	$b = 94616^m$, log b	4.97596
C = 61° 15′ 50″, log sin c...	9.94292	C = 50° 10′ 20″, log sin c.....	9.88535
m for 36° L., log m	1.40491	m for 45° 15′ L., log m	1.40398
ε = 0″.07	8.22497	ε = 22″.61	1.35433

DISTRIBUTION OF ERROR.

In the primary series the errors are adjusted by the application of the method of least squares, as explained and exemplified in appendices Nos. 33 and 14, Annual Reports for 1854 and 1864; or distributed in proportion to the probable error of each angle as determined directly from the measures of the angles.

In the secondary triangulation the error is distributed as above, or in inverse proportion to the number of measures taken of each angle, or, in less important cases, equally among the three angles.

In triangles of the third order the error, as a rule, is equally distributed among the three angles.

ABSTRACTS AND COMPUTATIONS.

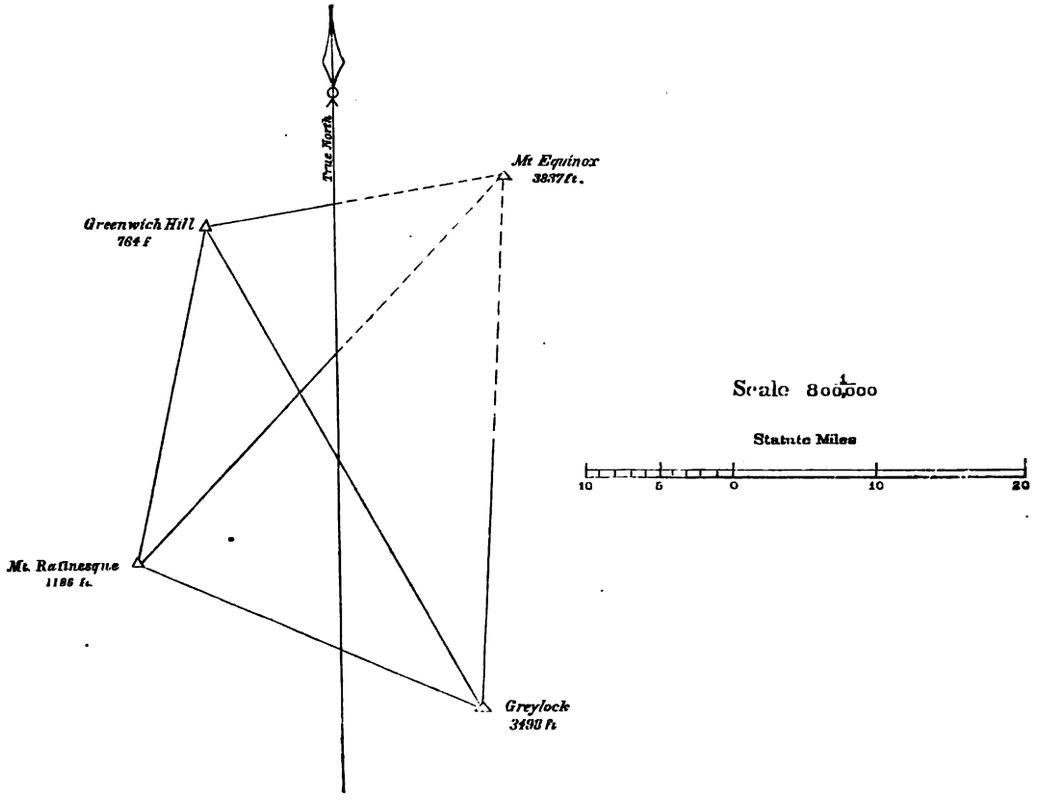
Form for abstract of angles.

Station, Mount Rafinesque, New York.—Instrument, Theod. No. 18.

Date.	A. M. or P. M.	Stations observed.	Tel.	No. of rep.	Angle.	Mean of D. & R.	Corr.	Mean angle.
	h. m.				° ′ ″	″	″	° ′ ″
Aug. 6	5 40 a. m	Greenwich Hill	D	6	32 48 57.6	.....	.....	.....
6	.....	Mount Equinox	R	6	59.2	58.4	.....	32 48 58.4
6	.....	do	D	6	57.2	.....	.....	.....
6	.....	do	R	6	57.4	57.3	.....	57.3
Aug. 7	5 00 p. m	do	D	6	57.8	.....	.....	.....
7	.....	do	R	6	59.0	58.4	*— 0.1	58.3
7	.....	do	D	6	56.8	.....	.....	.....
7	.....	do	R	6	57.2	57.0	*— 0.1	56.9
Aug. 9	6 00 a. m	do	D	6	58.3	.....	.....	.....
9	.....	do	R	6	59.5	58.9	.....	58.9
Aug. 10	6 10 p. m	do	D	6	57.3	.....	.....	.....
10	.....	do	R	8	58.4	57.9	.....	57.9
								32 48 57.95
Correction for eccentricity of instrument								+ 0.37
								32 48 58.32
Helderberg, &c., &c								
Greenwich								

*For phase.

The preceding form shows the application of the corrections for phase and eccentricity, and the one which follows, the spherical excess and distribution of error. It will be noticed that the station of Mount Equinox has not been occupied, and, hence, the angle is marked "concluded;" also, that in the arrangement of the triangles of the quadrilateral, the base being Greylock, (Mount Rafinesque,) the lines from the two unknown stations are entered according to their azimuth.



Form for computation of triangle sides.

SECONDARY QUADRILATERAL.

Denomination.		Observed angles.	Corr'n.	Spher'l angles.	Spher'l excess.	Plane angles and distances.	Logarithms.
						m.	
Greylock to Mount Rafinesque						40540.16	4.6078854
Greenwich Hill	76	39 43 31.63	+ 0.32	31".95	— 1.27	39 43 30.68	0.1944271
Greylock	67	37 01 16.33	+ 0.32	16" 65	— 1 27	37 01 15.39	9.7796731
Mount Rafinesque	62	103 15 14.88	+ 0.32	15".21	— 1.27	103 15 13.93	9.9832752
Greenwich Hill to Mount Rafinesque						38193.16	4.5819856
Greenwich Hill to Greylock						61743.00	4.7905877
Greylock to Mount Rafinesque							4.6078854
Mount Equinox	c			36.46	— 1.89	40 26 34.57	0.1879625
Greylock	99	69 07 12.65			— 1.89	69 07 10.76	9.9704986
Mount Rafinesque	65	70 26 16.56			— 1.89	70 26 14.67	9.9741783
Mount Equinox to Mount Rafinesque						58391.08	4.7663465
Mount Equinox to Greylock						58887.97	4.7700265
Mount Equinox to Greylock							4.7700265
Greenwich Hill	74	69 14 52.77		52.77	— 1.63	69 14 51.14	0.0291327
Mount Equinox	c				— 1.63	78 39 14.17	9.9914285
Greylock	67	32 05 56.32			— 1.63	32 05 54.69	9.7254026
Greenwich Hill to Greylock						61743.00	4.7908877
Greenwich Hill to Mount Equinox						33462.76	4.5245618
Mount Rafinesque to Greenwich Hill							4.5819856
Mount Equinox	c			40.36	— 1.02	38 12 39.32	0.2086194
Mount Rafinesque	74	32 48 58.32			— 1.02	32 48 57.30	9.7339526
Greenwich Hill	73	108 58 24.40			— 1.02	108 58 23.38	9.9757401
Mount Equinox to Greenwich Hill						33462.44	4.5245576
Mount Equinox to Mount Rafinesque						58390.90	4.7663451

COMPUTATIONS OF THE GEODETIC LATITUDE (L), LONGITUDE (M), AND AZIMUTH (Z).

The formulæ, tables, and examples for the computation of the geodetic latitudes, longitudes, and azimuths are given and fully explained in appendix No. 36, Coast Survey Annual Report for 1860.

For the sake of easy reference during the rapid and numerous computations of this character required from the assistant in the field, the signs, and their application, of the corrections depending on the relation of the given azimuth to the different quadrants, and on the algebraic signs of dL and dZ, are tabulated as follows:

Corrections to given Latitude.	
° °	° °
Z = 0 to 90	Z = 90 to 180
Z = 270 to 360	Z = 180 to 270
d L subtractive from L.	d L additive to L.
"	"
+ 1853.590	— 448.308
+ 22.760	+ .051
+ 0.084	+ .005
— 0.268	+ .000
$-dL = +1876.166$	$-dL = -448.252$

Corrections to given Longitude.
° °
Z = 0 to 90
Z = 90 to 180
d M additive to M.
° °
Z = 180 to 270
Z = 270 to 360
d M subtractive from M.

Corrections to given Azimuth.
° °
Z = 0 to 90
Z = 90 to 180
d Z subtractive from Z.
° °
Z = 180 to 270
Z = 270 to 360
d Z additive to Z.

The angles used in the above computations are the spherical angles taken from the computation of triangle sides.

SPIRIT-LEVELING.

The spirit-level is employed to determine the height, above mean tide, of the principal triangulation stations on the coast, to serve as bases for the trigonometrical leveling, and, indirectly, the co-efficient of refraction; and, also, the difference in height of the mean tide at different localities for use in the discussion of the tides.

To obtain the required degree of accuracy and confidence in the results, the following system has been adopted:

I. The line starts from a bench-mark on the coast, the height ofwhich above the mean tide has been determined by observations during not less than a semi-lunation.

II. The line is re-leveled in an opposite direction; or, in other words, the route is divided into short sections, and each section is leveled, forward and back, before proceeding to the next. The intermediate bench-marks supply intermediate checks.

III. The instrument and the method of observing and recording are so arranged as to secure the adjustment of the instrument or its equivalent, checks upon the leveling, and precision in the results.

The details of the above operations are given in the "general directions for running a line of levels." It is proposed here simply to refer to III.

The spirit-level, whether of the pivot or ordinary construction, has the diaphragm of its telescope provided with a reticule of two vertical and three horizontal wires.

The leveling-rod is carefully divided at the office into metres, decimetres, centimetres, and half-centimetres, and is provided with two small levels inserted in the rod, with which to preserve its verticality when held upon the foot-plate.

The foot-plate is of iron, triangular in shape, with a circular cavity in which the rounded extremity of the rod rests, and with three short legs, by means of which the plate can be firmly planted in the ground by a stamp of the foot. A light chain and ring is attached to the plate, by which the rodman can pull it up and carry it on to the next station.

Three leveling-rods and three foot-plates belong to the party, one set being held in reserve in case of accident to either of those in use.

The observer reads and records the heights on the rod crossed by the three wires, estimating the millimetres, or he can use a target, and a record of the readings be kept by both the observer and rodman. The condition of the level at every observation is entered in the record.

The instrument should be, theoretically, midway between the rods, and the distance from instrument to rod should rarely exceed 100 metres.

Before commencing the leveling, the value of the constants belonging to the particular instrument, and which are required for the reduction of the observations, are carefully determined.

CONSTANTS.

L.....	value of one division of level	3″.5
a_1 and a_2.....	angular distances between the wires.....	207″.01 and 198″.95
A	angular distance between extreme wires	405″.96
m.....	for reducing mean of three wires to middle wire..	2″.6867

To ascertain the angular distances, select a level spot of ground and measure off, with all possible care, distances from the instrument of 50, 100, 150, and 200 metres; adjust the instrument and read and record the heights on the rod crossed by the wires at each distance. This operation should be repeated at least four times for each distance.

Mean of four alternate readings at each distance.

50 metres.	100 metres.	150 metres.	200 metres.
m.	*m.*	*m.*	*m.*
0.0493	0.1007	0.1507	0.1997
.05033	.10067	.14967	.20067
0.0997	0.2013	0.3003	0.4003
.04833	.09766	.14300	.19233
0.1480	0.2990	0.4433	0.5927
.09866	.19833	.29267	.39300

Angular value of the distance between wires $= \frac{\text{difference in height}}{\text{D tang. } 1''}$, in which D represents the distance.

Difference = .05033	log Diff.	8.7018269	
D = 50m	co. log 50	8.3010300	
Co. log tang. 1″		5.3144250	
		2.3172819	207″.63=a_1,

and so on, for the twelve differences, resulting as follows:

a_1	a_2	A
″	″	″
207.63	199.38	407.00
207.65	201.44	409.08
205.81	196.64	402.45
206.96	198.35	405.31
207.01	198.95	405.96

An error of one second in the angular value of A would produce an error of one milimeter at 200 meters, and an error in the distance of 0.5 of a meter.

$$\text{Constant } m = \frac{207''.01 - 198''.95}{3} = -2''.6867$$

The upper wire, as seen in the telescope, is, in reality, the lower wire.

The usual adjustments of the instrument are made every morning and duly entered in the record, and are repeated during the day should a change from any cause be suspected.

I. For level, or to make the axis of the level parallel with the optical axis of the telescope.

This adjustment may be incomplete, but the record will give the data for determining the error.

Level.		Level reversed.	
E	O	E	O
d	*d*	*d*	*d*
7.7	7.3	9.5	5.5
6.2	8.8	8.0	7.0
7.0	8.0	8.8	6.2
6.97	8.03	8.77	6.23

$\frac{1''.80}{2} = +\,0''.9$ $\quad 0''.9 \times 3''.5 = +\,3''.15 =$ error of level $= l$.

II. For collimation, or to bring the middle horizontal wire and the middle of the two vertical wires in the optical axis of the telescope.

The record will also show whether this adjustment was perfect or imperfect. For instance:

Normal position.			Turned on axis, 180°.			Normal position.		
d	*d*	*m*	*d*	*d*	*m*	*d*	*d*	*m*
7.5	7.5	1.105	7.4	7.6	1.100	7.0	8.0	1.106
		1.150			1.145			1.150
8.0	7.0	1.194	9.3	5.7	1.189	7.0	8.0	1.195
		1.1497			1.1447			1.1503
Correction for inclination		+.0004			+.0013			+.0008
Reduction to mean of wires		−.0006			−.0006			−.0006
		1.1495			1.1454			1.1505
		1.1505			1.1500			
Mean = 1.1500					+0.0046			
					+0.0023 = error of collimation.			

III. For verticality of axis, or to make the vertical axis of the instrument perpendicular to the axis of the level.

Form of the field-record.

BACK SIGHT.								FORE SIGHT.							
Station.	Level. Eye end.	Level. Object end.	Level. Difference.	Readings on rod.	Correction for inclination.	Corrected height.	Distance between extreme wires.	Station.	Level. Eye end.	Level. Object end.	Level. Difference.	Readings on rod.	Correction for inclination.	Corrected height.	Distance between extreme wires.
	d	*d*		*m*		*m*			*d*	*d*		*m*		*m*	
Tidal B. M...	7.5	7.5		2.050				1.....	6.8	8.2		1.049			
				2.150								1.150			
	8.0	7.0		2.248					7.0	8.0		1.248			
	15.5	14.5	+1.0	2.1493	+ .0035	2.1528	.198		13.8	16.2	−2.4	1.1490	− .0047	1.1443	.199
			+0.5								−1.2				
1.....	6.1	8.9		1.049				2	7.5	7.5		1.048			
				1.099								1.098			
	6.5	8.5		1.148					9.0	6.0		1.148			
	12.6	17.4	−4.8	1.0987	− .0034	1.0953	.099		16.5	13.5	+3.0	1.0980	+ .0026	1.1006	.100
			−2.4								+1.5				
2.....	7.4	7.6		1.150				3	7.0	8.0		2.120			
				1.180								2.160			
	9.3	5.7		1.208					7.0	8.0		2.200			
	16.7	13.3	+3.4	1.1793	+ .0016	1.1809	.058		14.0	16.0	−2.0	2.1600	− .0018	2.1588	.080
			+1.7								−1.0				
3.....	7.5	7.5		2.205				4	7.5	7.5		1.196			
				2.250								1.242			
	8.0	7.0		2.294					9.0	6.0		1.287			
	15.5	14.5	+1.0	2.2497	+ .0016	2.2513	.089		16.5	13.5	+3.0	1.2417	+ .0024	1.2441	.091
			+0.5								+1.5				
						6.6803	.444							5.6478	.470

We have now:

L = one division of the level = 3″.5

A = angular distance between extreme wires = 405″.96.

m = constant for reduction to middle wire = − 2″.6867.

l = error of level = + 3″.15.

c = error of collimation = + 0^{m}.0023.

d = observed distance between extreme wires.

D = observed distance from instrument to rod.

i = observed inclination.

And let

I = correction for inclination.

M = correction for middle wire.

C = correction for collimation.

R = correction for inequality of distance between back and fore sights.

Correction for inclination.

$$I = i\,d \cot A \tan 1''.$$

d	*d*		$\frac{d}{4}$		log's.
7.5	7.5	2.205	$\frac{15.5 - 14.5}{4}$	= + 1″.75 = *i*	0.24304
	2.250			,089 = *d*	8.94939
8.0	7.0	2.294		405.96 = A	2.70590
				tang 1″	4.68557
			correction = + .0004		6.58390

Correction to reduce mean of three wires to middle wire.

$M = d \cot A \tang m.$	log's.
$d =$ .089	8.94939
$A =$ 405″.96	2.70590
$m =$ −2.6867	5.11479
correction, = − .0006	6.77008

Correction for error of collimation.

$$c = \frac{c}{d} \cdot \left(\frac{\tang A}{\tang 1''} \cdot \frac{1}{L} \right) \text{ in level divisions.}$$

$c = +.0023$	log c	7.36173
$d =$ .089	co. log d	1.05061
$A =$ 405″.96	log tang A	7.29410
	co. log tang 1″	5.31443
$\frac{1}{L} =$ 0.2857	co. log L	9.45591
correction, = +2″.997		0.47678

We now collect the errors from all sources expressed in terms of divisions of the level, and, calling the sum *i*, or error of inclination, the correction is applied to the mean of the three readings in the record.

	″
Error of level, left in adjustment.....	+ 3.15
Error of inclination.....	+ 1.75
Error of collimation	+ 3.00
	$i =$ + 7.90

and $I = i\ d \cot A \tang 1'' =$ + 0.0016.

So long as the values of L and A remain unchanged, the preceding computations are made once for all by the construction of tables from which the corrections can be taken out by inspection.

Table I.—Distance, instrument to rod, with argument *d*.

Table II.—Reduction to middle wire, with argument *d*.

Table III.—Double entry. Correction for error of inclination, with arguments *i* and *d*.

Table IV—Double entry. Correction for collimation, with arguments *c* and *d*.

Correction for inequality of distance between the back and fore sights.

$$\text{Reduction to true horizon} = (1 - 2m)\frac{D^2}{2R}, \text{ in which}$$

$m =$ co-efficient of refraction .070

R = mean radius of earth $\log R = 6.8039618$

D = the distance $d \cot A$.

Reduction = $0^m.000000067532\ (d \cot. A)^2$.

The maximum distance from the instrument to rod being confined, as a rule, to 100 meters, the greatest probable inequality may be assumed at 30^m (.059) or 100^m (.197) for the back sight and 70^m (.138) for the fore sight, the correction for which would not amount to more than one-third of a millimeter. However small and inappreciable in a single case, an aggregate of such differences might have a sensible effect upon the difference of level. Let us suppose that the distance between two bench-marks was about six miles, divided into 5527 and 4127 meters, or, in other words, that the sum of the values entered in the record under the head of the extreme wires for the back sight was $10^m.879$ and for the fore sight $8^m.123$, and that the number of stations was sixty. In this case the difference on account of curvature and refraction would be 0.9127, which, divided by the number of stations, would give 0.0152 or 0.6 of an inch as a closely approximate correction, subtractive from the sum of the back sights. While such great inequalities in the distance between the back and fore sights are not probable, it would be advisable to make out a table of double entry, in which d_1 represents the smaller distance and d the inequality, by the following formula. The correc-

tion for the day's work or between any two bench-marks could then be taken out by inspection. The quantity taken out must still be divided by the number of stations.

$$\text{Correction} = 0^{m}.000000067532\ (d^2 + 2dd_1).$$

TRIGONOMETRICAL LEVELING.

Differences of height are determined by the measurement at one station, by means of a vertical circle, of the double zenith distance of the signal at each of the other stations.

Another method is to measure, by means of a micrometer inserted in the eye-piece of the telescope of the theodolite employed for horizontal angles, the differences in altitude between the different stations, expressed in minutes and seconds of arc, in connection with one or more of the stations, or a reference mark, the absolute height of which, or its zenith distance, has been obtained.

A similar series of observations being taken at each successive station, and assuming that we start from one the exact height of which above mean tide has been determined by the spirit-level, we obtain checks, or different values for the elevations, rarely exceeding 3″ to 5″ in arc, and these errors are finally assigned to their probable true place by the method of least squares.

The state of the level at the commencement and end of the observation, and the hour and minute at which it was made, are entered in the record, and also the observations made to determine the value, in seconds of arc, of one turn of the micrometer screw.

The height of the telescope above the ground at each station occupied, and of all the signals or objects observed upon, should be carefully measured and made part of the record.

The following example of the abstract for vertical angles will show the data which are indispensable, and, incidentally, the fact that by reducing, at the outset, the observed zenith distance to the ground at each station, some labor and possible complication are saved in the final computations. The correction for error of level and of inclination, taken out from a table constructed for the purpose, is applied in the record, and this corrected zenith distance is called in the abstract the observed zenith distance.

To convert the difference in meters between the height of the telescope and of the object observed upon into seconds of arc:

Seconds of arc $= \frac{r}{K \sin 1''}$, in which r represents the difference in meters, and K the distance between the two stations.

Vertical angles.

Station, Helderberg.—Instrument, Repsold, No. 17.

MOUNT RAFINESQUE.

Date.	Hour of day.	Object observed upon.	Above station marks.		Correction.		Observed zenith distances.			No. of reps.	Zenith distances between station marks.		
			Object.	Telescope.	Feet.	Seconds.							
1876.	*h. m.*		*Feet.*	*Feet.*	*Feet.*	+	°	′	″		°	′	″
September 21 ..	2 30 p. m....	Apex of tripod	22.15	6.0	16.15	27.5	90	25	49.7	5	90	26	17.2
21...	4 40 p. m...	.. do	22.15	6.0	16.15	27.5			43.9	5			11.4
25...	4 20 p. m...	Center of cone..	43.67	6.0	37.67	64.0			17.0	5			21.0
24...	3 10 p. m...	...do	43.67	6.0	37 67	64.0			08.5	5			12.5
25...	11 a. m.....	.. do	43.67	6.0	37.67	64.0			01.1	4			05.1
25...	1 15 p. m ...	...do	43.67	6.0	37.67	64.0			20.6	5			24.6
25...	2 05 p. m....	....do	43.67	6.0	37.67	64.0			20.7	5			24.7
25...	2 45 p. m...	.. do	43.67	6.0	37.67	64.0			21.7	5			25.7
28...	2 50 p. m....	....do	43.67	6.0	37.67	64.0			19.8	5			23.8
29...	2 50 p. m....	...do	43.67	6.0	37.67	64.0			27.9	5			31.9
October 4...	3 05 p. m....	...do	43.67	6.0	37.67	64.0			12.9	5			16.9
6...	2 35 p. m...	Apex of tripod	22.15	6.0	16.15	27.5			47.0	5			14.5
Mean	.....	.....	.....	.....	.....	.....	.....			.....	90	26	18.3

* The results in brackets were obtained within so short a period of time, one from the other, that they must be considered in each case as a single result, or expression of the atmospheric refraction.

Form for micrometrical differences of height.

Station, Flat Top, Va.—Date, June 24, 1876.—Observer, A, T. M.—Instrument, 14-inch Würdemann Theod., No. 10.

Object observed.	Time.	Micr.	Level.		Remarks.
			O	E	
SERIES IV.	*h. m.*	*t.*	*d.*	*d.*	
Tobacco Row, crotch ..... v. unst..	0 28 p. m.	6.82	13.8	11.2	Crotch 22.82 feet above △.
Spear, heliotrope..... ... .ft..	.....	1.42	14.3	10.4	Heliotrope, 11.33 feet above △.
Smith, ground	.....	0.38	12.3	12.8	
S. W. Peak, rock ..... v. unst..	.....	2.14	13.2	11.8	
Cahas, ground .....v. ft..	.....	14.03	13.2	11.8	
Buffalo, top of trees..... v. ft..	.....	12.08	13.3	11.7	Secondary objects. Tops 30 feet above ground.
Sugarloaf, top of trees ..... dst..	.....	3.92	13.3	11.8	Do.
Tobacco Row, crotch ....v. unst..	0 45 p. m.	6.83	13.7	11.3	Weather clear. Wind light, S. W. Bar. 26.82 in. Ther. 72°.7 F. Telescope above ground, △ 5.99 feet.
SERIES V.					
Etc., etc.					

Reciprocal zenith distances measured at any two stations at the same moment of time, or under the same supposed condition of the atmosphere, give the best results. When reciprocal, but not simultaneous, the observations should be made on different days, as in the case of horizontal angles, in order to obtain, as far as possible, a mean value of the difference between the respective angles and an average value of the refraction. The same care should be taken when the zenith distance is measured at one station only.

Experience has proved that the refraction is greater and more variable at sunrise than at any other hour of the day; that it gradually diminishes, in both respects, until 9 or 10 a. m.; that between those hours and 3½ p. m. it is comparatively stationary, and from 3½ p. m. to sunset it increases in amount and variation, being the greatest during the night. The best period for observing, therefore, is between 9 a. m. and 3½ p. m., and the worst at sunrise and sunset.

The condition of the atmosphere and the relative refraction may be so different at stations situated more than twenty miles apart, that, as a general rule, the difference of level determined even by reciprocal observations cannot be relied upon for the desired degree of accuracy at distances greater than about twenty miles, unless a very large number of measurements have been made under the most favorable circumstances. The higher the elevations the more reliable the results.

When a station is occupied for vertical observations, the zenith distance of every signal in sight should be measured, so as to have more than one check for each altitude. A number of such measures and differences of level should be so combined, by the method of least squares, as to give the most probable values for the respective altitudes, as well as for the co-efficient of refraction for the period of observation.

I.—*By reciprocal zenith distances, simultaneous or not.*

Let Z, Z′ = the measured zenith distances of the telescopes at the two stations.
K = distance, in meters, between the two stations.
R = radius of curvature of the arc joining the two stations.
C = angle at the earth's center, subtended by the arc.
h, h' = heights of the two stations above mean tide.

$$C = \frac{K}{R \sin 1''} \qquad h - h' = \frac{K \sin \frac{1}{2}(Z' - Z)}{\cos \frac{1}{2}(Z' - Z + C)}$$

The value of R, or of $\frac{1}{R \sin 1''}$, (which depends on the mean latitude of the two stations and the angle made by the arc with the meridian,) may be computed for different latitudes, and for angles varying from 0° to 90°, as in the following table, based upon Clarke's constants.

Table of logarithms of radius of curvature.

	Azimuth.	LATITUDE.						
		24°	26°	28°	30°	32°	34°	36°
	°							
Meridian.....	0	6. 802479	6. 802597	6. 802722	6. 802852	6. 802988	6. 803129	6. 803274
	5	2498	2615	2739	2869	3004	3145	3289
	10	2553	2669	2791	2919	3052	3190	3332
	15	2644	2756	2875	3000	3130	3265	3404
	20	2766	2875	2990	3111	3236	3366	3500
	30	3093	3192	3296	3405	3518	3636	3757
	40	3496	3580	3671	3766	3864	3967	4072
	50	3923	3994	4070	4150	4233	4319	4407
	60	4325	4384	4446	4512	4580	4650	4723
	70	4653	4702	4753	4807	4863	4921	4980
	75	4776	4822	4869	4918	1969	5022	5076
	80	4867	4909	4953	4999	5047	5097	5148
	85	4923	4963	5006	5049	5096	5143	5192
Perpendicular	90	6. 804942	6. 804981	6. 805023	6. 805066	6. 805112	6. 805159	6. 805207
		38°	40°	42°	44°	46°	48°	50°
Meridian.....	0	6. 803422	6. 803573	6. 803726	6. 803880	6. 804035	6. 804189	6. 804342
	5	3436	3586	3739	3892	4045	4199	4351
	10	3478	3626	3775	3926	4077	4228	4378
	15	3546	3690	3835	3982	4130	4277	4423
	20	3637	3776	3917	4059	4201	4343	4484
	30	3880	4006	4133	4262	4391	4519	4647
	40	4179	4289	4400	4511	4623	4735	4846
	50	4498	4590	4683	4777	4871	4965	5058
	60	4797	4873	4949	5025	5104	5181	5257
	70	5041	5104	5166	5229	5293	5357	5420
	75	5133	5190	5248	5307	5364	5423	5481
	80	5201	5254	5308	5363	5417	5472	5526
	85	5242	5294	5345	5397	5450	5502	5554
Perpendicular	90	6. 805256	6. 805307	6. 805358	6. 805409	6. 805460	6. 805512	6. 805563

Example.

K=23931^{m}.6.....distance between two stations, Santa Cruz and Mount Bache, California.
Z =87° 35′ 01″.06 ..observed at Santa Cruz station, reduced to ground at Mount Bache.
Z′=92° 35′ 34″.20 ..observed at Mount Bache, reduced to ground at Santa Cruz station.
L =37° 02′ .....mean latitude of the two stations.
angle=51° 55′ .....angle made by line with the meridian.

Computation of h—h′.

			°	′	″		
log K.....	4.3790	Z′—Z.....	5	00	33.14	log K.....	4.3790
co. log R sin 1″..	8.5101	½ (Z′—Z).....	2	30	16.57	log sin ½ (Z′—Z).....	8.6405
		Z′—Z+C	5	13	28.06	.co. log cos ½ (Z′—Z+C).	0.0004
log C.....	2.8891	½ (Z′—Z+C)...	2	36	44.03		
C =	774.56						3.0199

	m
Difference in height.....	1046.90
Santa Cruz station above mean tide—by spirit level.....	108.87
Mount Bache above mean tide.....	1155.77

II.—*By the zenith distance measured at one station.*

Let Z=the measured zenith distance of the signal or object.
K=the distance between the two stations, in meters.
m=the co-efficient of refraction.

$$C=\frac{K}{R \sin 1''}$$

dh=difference in height between the two stations.

$$dh=\frac{K \cos (Z+mC-\frac{1}{2}C)}{\sin (Z+mC-C)}$$

Example.

Z=87° 07′ 18″.8 ..... observed at Farmington upon crotch at Mount Blue.
K=15519^m ..... distance between Farmington and Mount Blue.
m=0.071 ..... co-efficient of refraction.
L=44° 42′ ..... mean latitude of the two stations.
angle=65° 44′ ..... angle made by line with meridian.
Telescope above ground=2^m.2. Crotch above ground=4^m.4.

					° ′ ″		
log K	4.1907	C	501″.2	Z+mC	87 07 54.4	log K	4.19086
table	8.5093	m	0″.071	Z+mC−½ C..	87 03 43.8	log cos	8.70971
		mC	35″.6	Z+mC−C ...	86 59 33.2	co. log sin ..	0.00060
log C	2.7000						2.90117

dh—between telescope and crotch	^{m} 796.47
Telescope above ground	+2.20
Crotch above ground	—4.40
dh—between ground at Farmington and Mount Blue	794.27
Ground at Farmington above mean tide	181.20
Ground at Mount Blue above mean tide	975.47

III.—*By the observed zenith distance of the sea horizon.*

Z=the measured zenith distance.
R=radius of curvature of arc.
m=co-efficient of refraction=0.078.

$$h=\frac{R}{2(1-m)^2}\tan^2 (Z-90°)$$

Example.

Z=90° 24′ 19″.1
L=41° 15′
angle=15°
R=6.8038 from table.

Telescope above ground	^{m} 1.67		
State of tide—below half-tide	0.30		
(1−m)=(1−0.078)=0.922, log (0.922)^2	9.92946		
log 2	0.30103		
		log R	6.8038
log 2 (1−m)^2	0.23049	.....	0.2305
$\log \frac{R}{2(1-m)^2}$			6.5733
(Z−90°)=24′ 19″, log tang^2 (24′ 19″)			5.6994
			2.2727

	m
Diff. of height between telescope and sea horizon.....	187.37
Telescope above ground	—1.67
Reduction for state of tide	—0.30
Height of station above mean tide.....	185.40

IV.—*By observed angles of elevation or depression.*

A=the observed angle, expressed in seconds.
K=the distance, in meters, between the two stations.
constant=0.00000485 log constant=4.68574.
constant=0.0000000667 log constant=2.82413.

$$dh=0.00000485\ K\ A+0.0000000667\ K^2.$$

This formula gives the difference in height between stations not more than ten or fifteen miles apart, with a probable error less than the uncertainty in the co-efficient of refraction.

Example.

A=2° 52′ 41″.2 = 10361″.2=angle of elevation.
K=15519^m=distance between the two stations.

log A.....	4.01541				
log K.....	4.19086		log K^2.....	8.38172	
log constant.....	4.68574		log constant	2.82413	
	2.89201....	m 779.85		1.20585 ...	m 16.06
		16.06			
		dh=795.91			

THE CO-EFFICIENT OF REFRACTION.

The co-efficient of refraction, or proportion of the intercepted arc, is determined from the observed zenith distances of two stations, the relative altitudes of which have been determined by the spirit-level; or, from reciprocal zenith distances, simultaneous or not, under the assumption that the mean of a number of observations taken under favorable conditions will eliminate the difference of refraction which is found to exist, even at the same moment, at two stations a few miles apart. Such a co-efficient may be established for the level of the sea, or for high elevations, or for lines over water or over land. As, however, the difference of height, deduced from trigonometrical leveling, depends upon the co-efficient multiplied by the square of the distance, it is evident that the longer the line, the greater would be the error caused by any uncertainty in the co-efficient or actual refraction; and that, consequently, there is a limit to the distance for which any assumed mean value of the refraction can be depended on for accurate results.

The average value of the co-efficient from the Coast Survey observations is,

across parts of the sea near the coast.....	0.078
between primary stations	0.071
in the interior of the country, about	0.065

To determine the co-efficient of refraction from reciprocal zenith distances.

C=angle at earth's center subtended by arc.
F=angle of refraction.
m=co-efficient of refraction.

$$C=\frac{K}{R\ \sin 1''} \qquad F=\frac{C}{2}-\tfrac{1}{2}\,(Z'+Z-180^\circ) \qquad m=\frac{F}{C}$$

Example.

Adopting the observations and corrections in the example to I, and leaving out, as in that case, the very small corrections depending on the height of the stations above the mean surface of the sphere, we have,

	′ ″	″	log's.
$Z+Z'-180°$	10 35.26	F=69.7	1.8432328
$\frac{1}{2}(Z+Z'-180°)$	5 17.63	C=774.66	2.8891111
$\frac{C}{2}$	6 27.33	$m = 0.089975$	8.9541217
F	1 09.70		

To determine the co-efficient from the zenith distance observed at one station, when the altitudes of the two stations above half-tide, or their difference in height, have been determined by the leveling-instrument.

Compute the true zenith distances, Z'_o and Z_o, of the two given points, and the difference between the true and the observed zenith distance will be the angle of refraction, F; and $m = \frac{F}{C}$

$$\tfrac{1}{2}(Z'_o+Z_o) = 90° + \frac{C}{2}$$

$$\tfrac{1}{2}(Z'_o-Z_o) = \tan^{-1}\left(\frac{h'-h}{K}\left\{1-\frac{h'+h}{2R}-\frac{K^2}{12R^2}\right\}\right)$$

Example.

Adopting the data afforded by the example to II, we have,

$h'+h = 975^m.47$, supposed to be determined by leveling-instrument.
$h'-h = 794^m.27$, supposed to be determined by leveling-instrument.
Z = observed Z. D.+correction for height of crotch above telescope.
$Z = 87° 07' 18''.8+29''.2 = 87° 07' 48''$
$K = 15519^m$ $\log K = 4.1908637$
$C = 501''.2 = 8' 21''.2$; and $\frac{C}{2} = 4' 10''.6$

$\log R^2 = 3.6102$	$\log K^2 = 8.3817$	
$\log 12 = 1.0792$	4.6894	
4.6894	3.6923	-0.00000049
$\log R = 6.8051$	$\log h'+h = 2.9892$	
$\log 2 = 0.3010$	$\log 2R = 7.1061$	
7.1061	5.8831	-0.00007641
		-0.00007690
		0.99992310

$\log 0.99992310 = 9.9999666$

$\log h'-h = 2.8999682$
$\log K = 4.1908637$
$\log \frac{h'-h}{K} = 8.7091045$

$\log\left(1-\frac{h'+h}{2R}-\frac{K^2}{12R^2}\right) = 9.9999666$

$\log \tan = 8.7090711$
$2° 55' 46''.7$

	° ′ ″	° ′ ″
$\frac{1}{2}(Z'_o+Z_o) = 90+4\ 10.6$	=	90 04 10.6
$\frac{1}{2}(Z'_o-Z_o)$	=	2 55 46.7
true Z'_o	=	92 59 57.3
true Z_o	=	87 08 23.9
observed Z	=	87 07 48.0

F =	35.9	$\log F = 1.5550944$
C =	501.2	$\log C = 2.7000111$
$m = \frac{F}{C}$	= 0.0716	$\log m = 8.8550833$

THE THREE-POINT PROBLEM.

If three points, forming a triangle of which the sides and angles are known, or can be computed, be visible from a fourth point P, it is required to determine the position of P.

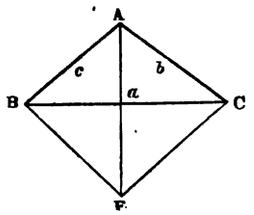
Set up the theodolite at P, and measure the two angles subtended by any two of the given sides.

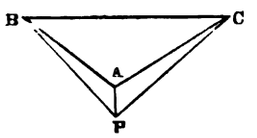
This problem is of use in cases where the regular triangulation having been completed, additional points are required for the topographical survey, or are needed for special service. The angles should be carefully measured, and, in the computations, the logarithms should be carried to seven places of decimals.

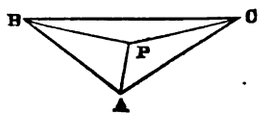
Three cases of its application are given, as in others, such as when P falls upon one or the other of the sides of the known triangle, or on the prolongation of either, the case resolves itself into the solution of a simple triangle with one side and the angles given; or the problem is indeterminate, as when P is situated on the circumference of the circle passing through the three known points—a contingency which rarely occurs.

Example for each of the three cases.

Given the side	a=11204.5	Angle observed	A P C=P′
Given the side	b= 7289.0	Angle observed	A P B=P″
Given the side	c= 6273.8	To find	A B P=x
Given the angle	A=111° 10′ 54″	To find	A C P=y







P′	50° 06′ 12″	49° 47′ 20″	104° 00′ 00″
P″	43° 50′ 38″	44° 09′ 30″	100° 20′ 00″
S	180°−½ (A+P′+P″)	½ (A−P′−P″)	180°−½ (A+P′+P″)
S	77° 26′ 08″	8° 37′ 02″	22° 14′ 33″

$$\tan Z=\frac{c \sin P'}{b \sin P''} \qquad \epsilon=\tfrac{1}{2}(x-y) \qquad \tan \epsilon=\cot (Z+45°) \tan S$$

x=S+ϵ, y=S−ϵ, but if tan ϵ be negative, then x=S−ϵ, y=S+ϵ

Computation.

log c	3.7975307	3.7975397	3.7975307
log sin P′	9.8849100	9.8829061	9.9869041
co. log b	6.1373320	6.1373320	6.1373320
log co. sin P″	0.1594574	0.1569894	0.0071016
log tan Z	9.9792301	9.9747583	9.9288684
Z	43° 37′ 49″.6	43° 20′ 09″.2	40° 19′ 43″.3
log cot (Z+45°)	8.3785397	8.4631818	8.9122794
log tan S	0.6519386	9.1805366	9.6116787
log tan ϵ	9.0304783	7.6437184	8.5239581
ϵ	6° 07′ 21″.7	0° 15′ 08″.1	1° 54′ 50″.04
S	77° 26′ 08″.0	8° 37′ 02″.0	22° 14′ 33″.00
x	83° 33′ 29″.7	8° 52′ 10″.1	24° 09′ 23″.00
y	71° 18′ 46″.3	8° 21′ 53″.9	20° 19′ 43″.00
Hence,		Hence,	Hence,
P A B	52° 35′ 52″.3	126° 58′ 19″.9	55° 30′ 37″.00
P A C	58° 35′ 01″.7	121° 50′ 46″.1	55° 40′ 17″.00

As all the angles and a side in each triangle are now known, the other sides, or the distances from P to the three given points, can be readily computed.

	m		m		m
P B	7194.87	P B	7194.94	P B	5256.29
P A	8999.89	P A	1388.54	P A	2609.75
P C	8107.98	P C	8107.91	P C	6203.63
P A	8999.89	P A	1388.54	P A	2609.75

The results are verified when both triangles give the same value for the line P A.

For the problem and an example by Schott, of determining a position by angles observed upon a number of given stations, see page 116, Coast Survey Report, 1864.

RECTANGULAR CO-ORDINATES ON A PLANE PROJECTION.

The method of plotting the position of trigonometrical points by rectangular co-ordinates is occasionally adopted in the field, and consists in referring the points of the triangulation to two straight lines, intersecting each other at right angles, called the co-ordinate axes. If O be the point of intersection, or origin of the axes, then y will be the axis of ordinates, x the axis of abscissæ, and Aa, Oa, or y, x, the rectangular co-ordinates of a given point A.

Should the true meridian be known, let that be the axis of ordinates, and the point of intersection a trigonometrical point; if not known, adopt, as the axis of abscissæ, a side of one of the triangles, which, if extended in one or both directions, would pass through the center of the triangulation, or as near thereto as possible. The triangulation point at one or the other end of this line, as may be preferred, will be, therefore, the origin of ordinates. Whatever may be the direction of the line, assume it, for the present purpose, to be a meridian, and count the azimuths from south to west, as in the L, M, Z computations.

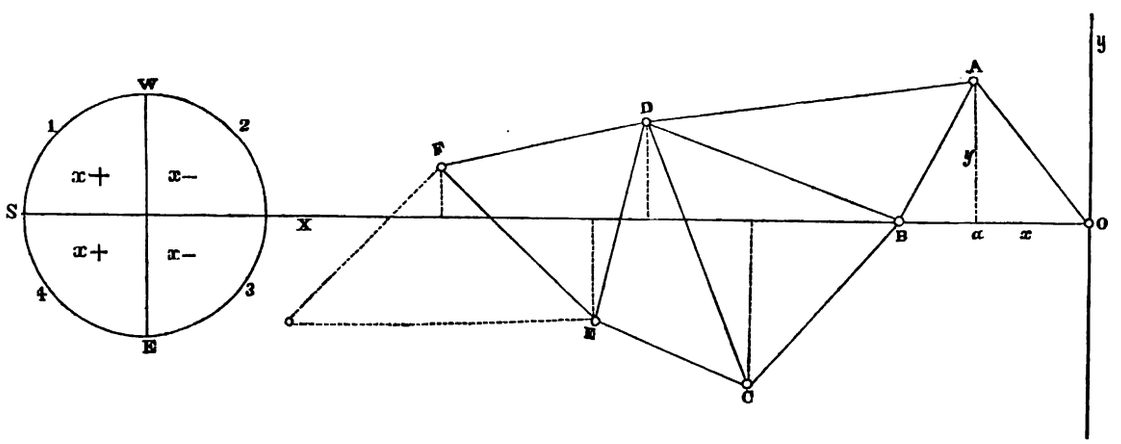
The values of the rectangular co-ordinates, y and x, are obtained by multiplying the triangle side by the sine and cosine of its azimuth.

The abscissa, x, will be additive when the azimuth is in the first and fourth quadrants, and subtractive when in the second and third.

The computed ordinate y will be subtractive from or additive to the last ordinate, according as the azimuth converges to or diverges from the axis of abscissæ.

If the ordinates on one side of the axis of abscissæ be regarded as positive, those on the other side will be negative.

Should the line adopted pass through the chain of triangles, as in the following example, having trigonometrical points on both sides, compute the rectangular co-ordinates for the points on one side, and then for those on the other, and the computation will be verified when the sum of the abscissæ, on reaching the terminal point, shall be the same by both series.



	Denomination.	Plane angles and distances.	Logarithms.		Denomination.	Plane angles and distances.	Logarithms.
	O to B	5073m.68	3. 7053234		D to C		3. 8889246
A	Three Sisters	65° 27′ 55″.3	0. 0410968	E	Persimmon Point	99° 38′ 59″.1	0. 0061888
O	North End	51° 30′ 37″.9	9. 8936078	D	South Base	36° 00′ 15″.2	9. 7692627
B	Ragged Island	63° 01′ 26″.8	9. 9499740	C	Caffee's Point	44° 20′ 45″.7	9. 8444710
	A to B	4365m.44	3. 6400280		E to C	4617m.17	3. 6643761
	A to O	4970m.43	3. 6963942		E to D	5490m.15	3. 7395844
	A to B		3. 6400280		D to E		3. 7395844
D	South Base	28° 29′ 34″.2	0. 3214372	F	North Base	53° 57′ 19″.6	0. 0922879
A	Three Sisters	53° 19′ 39″.9	9. 9042095	D	South Base	64° 48′ 09″.9	9. 9565754
B	Ragged Island	98° 10′ 45″.9	9. 9955595	E	Persimmon Point	61° 14′ 30″.5	9. 9428302
	D to B	7339m.64	9. 8656747		F to E	6143m.95	3. 7884477
	D to A	7194m.90	3. 9570247		F to D	5252m.54	3. 7747025
	D to B		3. 8656747				
C	Caffee's Point	62° 21′ 23″.7	0. 0526387				
D	South Base	48° 29′ 05″.4	9. 8743544				
B	Ragged Island	69° 09′ 30″.9	9. 9706112				
	C to B	6203m.94	3. 7926678				
	C to D	7743m.27	3. 8889246				

	° ′ ″			° ′ ″	
Z O to B	00 00 00.0		Z B to O	180 00 00.0	
∠ B & A	51 30 37.9	log's.	∠ O & A	63 01 26.8	
Z O to A	51 30 37.9	sin 9.8936078	∠ A & D	98 10 45.9	
		D 3.6963942	∠ D & C	69 09 30.9	
		cos 9.7940492		50 21 43.6	
		3.5900020	Z B to C	309 38 16.4	sin 9.8865424
y	*x*	3.4904434			D 3.7926678
+3890.47	3093.45				cos 9.8047694
A			*y*	*x*	3.6792102
			0000.00	5073.68	3.5974372
			—4777.60	3957.65	
			—4777.60	9031.33	
			C		
	° ′ ″			° ′ ″	
Z A to O	231 30 37.9		Z C to B	129 38 16.4+	
∠ O & B	65 27 55.3		∠ B & D	62 21 23.7—	
∠ B & D	53 19 39.9		∠ D & E	44 20 45.7—	
Z A to D	350 18 13.1	log's. sin 9.2264111	Z C to E	22 56 07.0	log's. sin 9.5907205
		D 3.9570247			D 3.6643761
		cos 9.9937510			cos 9.9642340
		3.1834358			3.2550966
y	*x*	3.9507757	*y*	*x*	3.6286101
+3890.47	3093.45		—4777.60	9031.33	
—1525.58	8928.44		—1799.27	4252.16	
+2364.89	12021.89		—2978.33	13283.49	
D			E		
	° ′ ″			° ′ ″	
Z D to A	170 18 13.1		Z E to C	202 56 07.0+	
∠ A & B	28 29 34.2		∠ C & D	99 38 59.1—	
∠ B & C	48 29 05.4		∠ D & F	61 14 30.5—	
∠ C & E	36 00 15.2				
∠ E & F	64 48 09.9		Z E to F	42 02 37.4	log's. sin 9.8258767
Z D to F	348 05 17.8	log's. sin 9.3147189			D 3.7884477
		D 3.7747025			cos 9.8707748
		cos 9.9905460			3.6143244
		3.0894214	*y*	*x*	3.6592225
y	*x*	3.7652485	—2978.33	13283.49	
+2364.89	12021.89		+4114.57	4562.71	
—1228.63	5834.36		+1136.24	17846.20	
+1136.26	17846.25		F		
F					
&c.		&c.	&c.		&c.

MEASUREMENT OF SUBSIDIARY BASE-LINES.

The Coast Survey annual report for 1854 contains a full description of the compensating base apparatus employed in the measurement of the primary bases on the Atlantic Coast, and the report for 1857 a general description of the sliding-contact apparatus for subsidiary or intermediate bases. These intermediate lines are required either as checks upon the series of small triangles extending along the low coasts south of the Delaware, or for the determination of the distance and direction between two places which cannot be connected by triangulation except at very great expense and probable risk of accuracy. In the latter case, a succession of lines and their deviation in direction are measured, and, if the operation is carefully conducted, and the lines on the beach are not less than two miles in length, the results, when compared with distances from the primary bases, will not fail to be satisfactory.

To explain fully the different successive operations connected with the measurement of a subsidiary line, it may be, perhaps, the best plan to present an abstract of the report of the measurements made by the compiler on the Virginia Coast, south of Cape Henry, in 1867.

Base apparatus.—The assistant is referred to the description of the apparatus and the mode of using it as given in the Coast Survey annual report for 1857, with the remark that the defects in the original, as first employed in the third measurement of the English base on Hounslow Heath, in 1784, have been successfully remedied. In addition to those important improvements, the rods employed in 1867 were six meters in length instead of four; the level of the sector was made more delicate, and on each of the forward trestles a roller was placed to facilitate the movement of the bar, forward and back, during its final adjustment.

Length of rods.—A comparison of the two rods with the standard six-meter bar should be made at the office in Washington, both before and after they are used in the field, and, if no accident has occurred to either during the interval, the mean of the two comparisons should be adopted as their respective values.

In deducing the length of rod from comparisons with the standard six-meter bar No. 2, the following data may be employed:

Length of standard bar No. 2 at 32° F.	$5^m.99998233$
One division of the scale of comparator, $\frac{1}{1463.0}$ of an inch	$0^m.00000174$
Co-efficient of expansion for F. scale	$0^m.00000641$
Thermometer attached to standard—too high	—0°.7
Thermometers attached to rods ± (in this case correct.)	

Comparisons made at the Coast Survey Office, August 10, 1867.

Standard No. 2.		Rod No. 1.		Standard No. 2.		Rod No. 2.	
Therm.	Divisions.	Therm.	Divisions.	Therm.	Divisions.	Therm.	Divisions.
°		°		°		°	
77.3	+21	76.0	−10	75.3	+ 1	74.0	+7
78.0	+15	76.4	+41	76.0	+ 8	74.5	−3
78.5	+18	77.0	+55	77.0	+13	75.0 (74°.83)	+3 (+1)
77.93	+18	76.47	+28.67	76.1	+ 7.33	75.0	+3
− 0.70		77.23	+18.00	− 0.7		74.41	+4.00
77.23		+.76	+10.67	75.4		75.40	+7.33
						+ 0.99	−3.33

Computation of length of rod No. 1.

	m
$+0°.76 \times 0.00000641 \times 6^m$	+0.00002923
$+10^d.67 \times 0.00000174$	+0.00001857
at 77°.23, No. 1, longer than st'd.	+0.00004780
at 77°.23, standard No. 2	6.00172188
at 77°.23, rod No. 1	6.00176968
at 75°.00, rod No. 1	6.00168391

Computation of length of rod No. 2.

	m
$+0°.99 \times 0.00000641 \times 6^m$	+0.00003808
$-3^d.33 \times 0.00000174$	−0.00000579
at 75°.4, No. 2, longer than st'd.	+0.00003229
at 75°.4, standard No. 2	6.00165149
at 75°.4, rod No. 2	6.00168378
at 75°.0, rod No. 2	6.00166840

Similar comparisons were made in the month of November following, after the return of the apparatus from the field.

	Rod No. 1.	Rod No. 2.
August 10, 1867, length at 75° F.....	6m.00168391	6m.00166840
November 21, 1867, length at 75° F.....	6m.00168692	6m.00154393
Mean adopted for the measurement.....	6m.00168541	6m.00160616

Instruments, &c., and organization of party.—There will be required one 12-inch theodolite, two small transits, a leveling instrument, telescope, 20-meter chain, meter-scale and extension dividers, a coil of iron wire, one-eighth of an inch in diameter and 70 meters in length, and a spring balance.

One assistant, to make the contact, give the signals, &c.

One aid, to align the bars, using a transit.

One aid, to record the inclination, temperature, and number of the bar, and, when the measurement halts or stops for the day, to transfer the end of rod to copper tack in stub, employing, for this purpose, the other transit.

Two men, to carry the bar.

Two men, to pick up the trestles, carry them forward, adjust them in line, and level them.

One man, to attend the aid in charge of alignment, bring up instrument, &c.

One man, to keep up the transfer transit, and to be provided with stub, axe, and copper tack for an emergency, and to assist generally.

Cart, horse, and driver, for the transportation of heavy wooden box, in which the bars are kept when not in use; of water, stubs, spades, and tools, and of tent, in case of sudden storm.

The record book.—The record book is ruled and kept as in the following specimen pages, except that the remarks are applicable to the day's work, not to the particular number of bars. The left-hand page contains 20 bars or 120 meters, and, as the stubs of the preliminary measurement are driven at every 120 meters, the last number on each page should be an even number, and should coincide with a stub, in order that an error, in counting or numbering the bars, cannot be carried beyond the page. The thermometers, one for each rod, are read, and the temperature recorded for every ten bars, and oftener, if any unusual delay occurs, and always when the measurement stops, from any cause, whatever may be the number of the last bar. The inclinations are recorded for each rod, and the columns for the mean temperature and corrections for inclinations are filled up, as the opportunity may offer.

Measurement of the coast. September 22, 1867. VIRGINIA—SECTION IV.						Station B to station C. Clear. Wind, L., westerly.		
Time.	Whole number.	No. of Bar.	Temp.	Inclination. +	Inclination. —	Mean temperatures.	Correction for inclination.	Remarks.
h m			°	′	′	°		
11 25	161	1	77.5	42			.000448	Stopped for aligning transit
	162	2	77.5		17		0074	to advance.
	163	1			20		0102	
	164	2		7		76.875 × 10	0012	
	165	1		1.23		768.75	1749	
	166	2		21			0112	
	167	1			·48		0585	
	168	2			3		0002	
	169	1	76.5		5		0006	
11 41	170	2	76.0		19		0092	Stopped. Stub, copper tack,
12 43	171	1	74.5	13			0043	and cross-lines.
	172	2	74.5	9			0021	
	173	1			20		0102	
	174	2		19		74.25 × 10	0092	
	175	1		56		742.50	0796	
	176	2			37		0348	
	177	1			19		0092	
	178	2			57		0825	
	179	1	74.0	26			0172	
12 56	180	2	74.0		14		0050	Stopped for the day. Stub,
							.005723	copper tack, and cross-lines.

Preparation of the line.—The locality of the proposed base having been selected, either on the beach, adjoining sand plain, or hard land in the interior, the line is traced, and its direction adjusted to the least uneven ground and to avoid the tide, dunes, or hillocks, and then cleared of all minor obstructions.

Monuments.—After the line has been finally laid out, the monuments are erected. These may be merely the underground permanent marks, brought up level with the surface of the ground, and the terminus of the line marked thereon, the cap or upper block being reserved until the horizontal angles have been observed. The operation of ending the measurement at a given point is less liable to error and more economical of labor than the process of putting in a temporary mark, and then superseding it by a final one at some time after the measurement has been completed.

Alignment.—The base is aligned by setting up straight poles, prepared for the purpose, at about every half mile. This may be done by adjusting the transit over the mark at one end, fixing the cross-hairs on the signal at the other terminus, and by a system of signals to the aid, who is provided with a telescope, putting the poles in line, working in the direction of the instrument.

During the measurement an aid, with his transit, is stationed in the rear, never more than about one-fourth of a mile from the apparatus, and directs the alignment of the bars by movements of his arm. A small wand, about the size of a lead-pencil, and painted black, is firmly fixed, in a vertical position, at the forward end of each bar, directly over the rod, and when the bar has been approximately brought into position by the men who have it in charge, a signal is made, and the aid perfects the alignment.

Adjustment of the apparatus.—Before commencing the measurement, each bar is appropriately arranged upon trestles, and the knife-edge at one end and the center of the plane at the other are brought to the same level, by means of a leveling instrument placed about 20 meters from each end. The rod being level, the level and zero of the sector are adjusted to this condition of the rod. The same operation is gone through with after the measurement is completed, to ascertain if the relation between the sector and the rod has changed, or, in other words, if the readings of the sector have truly expressed the inclination of the rod; and, if not, the error. Half of this index-error is the correction to be applied to the recorded inclinations, though, if it is small, as in the following case, and the angles of elevation and depression are about equal in number and value, the error is not appreciable:

	Rod 1, index-error.	Rod 2, index-error.
Before measuring—station A, adjustment.....	0′.0	0′.0
After measurement—station B, compared	+1′.0	0′.0

Hence, for every + inclination recorded for rod No. 1, a correction of 0′.5 should be subtracted, and the same added to each − inclination. The sector of rod No. 2 had remained unchanged.

Preliminary measurement.—The preliminary measurement is made with an iron wire, about one-eighth of an inch in diameter, and 60 meters in length. This distance (10 bars) is measured off with the base apparatus, starting from the initial point, should the ground be favorable, and extending in the direction of the base-line. The terminus is marked by cross-lines drawn on the head of a copper tank driven in a stub sunk level with the surface of the ground. The inclinations and temperatures are recorded, in order that the exact distance may be computed. The wire, which has been previously uncoiled, straightened, and otherwise prepared, and also provided with a loop at each extremity, is then stretched between the two marks by a chain staff at the after end, and a weight of forty pounds applied to the other by means of a spring balance. In this condition, and after repeated trials, the measured distance is transferred to and marked on the wire by a fine line cut near each loop.

The wire is then drawn forward, stretched in line, the uniform weight of forty pounds applied, and at a given signal from the aid in charge of the after end, indicating the adjustment of the wire mark to the stub mark, the forward mark is transferred, by a pencil, to the planed surface of a small wooden bench, sunk nearly level with the ground. The pencilled line is then numbered, the wire carried forward, aligned and stretched as before, the after mark adjusted to line on bench, the forward mark transferred to another and similar bench, and so on to the terminus.

The bench, which is about one foot square, and provided with four short legs, is firmly planted in the ground by the pressure of the foot after the wire has been approximately adjusted for di-

rection and distance. Four or five of these benches will be required, one being always left a wire behind as a precaution against accident. Stubs are driven at every second wire, or 120 meters apart, to serve as tests of correctness in counting the bars during the final measurement. Each page of the record will commence and end at one of these stubs.

The end of the last wire, before reaching the terminus, should be marked by cross-lines on the usual copper tack and stub, so that when the final measurement reaches the mark a close comparison may be instituted, if desired, between the two measurements. As the last wire will either fall short of or go beyond the monument, the difference should be measured with the 20-meter chain, a tape-line, or meter-scale, according to the distance, and duly recorded with its appropriate sign.

The above operation requires very little more time than the ordinary chain-measurement, and is much more satisfactory. In order to show the accuracy with which such rapid measurements may be made on level ground, the two following comparisons are presented:

	m
By wire measurement. A to B = 82 wires × 6m.00164579 × 10	4921.3495
By base apparatus. A to B = 802 bars × 6m.00164579 − [excess = 0m.019] + [Rudie Creek = 107m.8408] + [temp. = 0m.2107] − [inclination = 0m.1681]	4921.1843
By wire. B to C = 71 wires × 6m.00164579 × 10	4261.1685
By apparatus. B to C = 710 bars × 6m.00164579 + [deficiency = 0.1016] + [temp = 0m.2200] − [inclination = 0m.1676]	4261.3225

The temperature of rods.—The thermometers attached to the apparatus are compared with the standard at the office, and the correction to be applied, if any, is marked on the scale. The temperature, as a rule, is read at every ten bars, and without withdrawing the thermometer from its position next to the rod. Every possible care should be taken to preserve an even or slowly changing temperature, and that the thermometer should express the temperature of the rod. The box in which the bars are deposited at the close of the day's work should be left in an east and west direction, so that one rod or one thermometer may not be found the next morning unduly heated by having been exposed to the morning sun; and should any great difference be noted between the two thermometers, a delay of half an hour is advisable to permit them to settle down to about the same degree. The measurement should stop when the temperature is over 100°.

Inclination of the rods.—The bars should not be allowed, for obvious reasons, to rest for any length of time on the trestles; and as rapidity in the measurement is in some degree essential to its success, no time should be lost during the operation in an attempt to keep the bars exactly level. The inclination should be observed while the assistant is perfecting the contact, and the correction therefor will answer every purpose. When the inclination exceeds 3° or 4°, or is beyond the range of the sector, a vertical offset becomes necessary. The transit is set up directly opposite to the end last adjusted, and very carefully leveled. The after-bar is then carried forward, placed on its trestles, leveled, and aligned, and then moved backward or forward, as the case may require, approximately by hand and finally by the slow-motion screw, until the knife-edge, adjusted for coincidence of line, is brought in the vertical of the plane of the fixed rod, under the direction of the aid in charge of the transit. When the contour of the ground shows that a vertical offset is unavoidable, the last bar shall be made as level as possible. Care should be taken, however, when approaching an elevation or depression in the ground, so to manage the inclination of the bars as to avoid or to have as few as possible of such offsets. A table containing the correction for each minute of inclination may be computed by the following formula:

R = length of rod ..... say $6^m.00164579$

θ = inclination in minutes ..... say $25'$

$$\text{correction} = \frac{\sin^2 1'}{2}\theta^2 R \qquad \text{constant} = \frac{\sin^2 1'}{2} \quad \log \text{constant } 2.626422$$

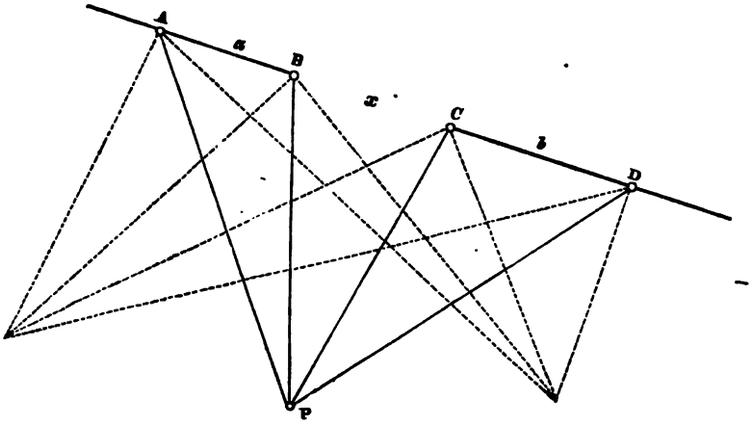
log constant	2.626422
log θ^2	2.795880
log R	0.778270
	6.200572

$0^m.00015870$

The distance across a creek, quicksand, &c.—When it is not possible to obtain the desired length of base-line without crossing a short distance in which the bars cannot be used or it would be

unsafe to depend upon them, as in the case of water, quicksand, boggy soil, or even a rapid succession of great irregularities in the ground, the unmeasured distance should be determined by the following method:

Let AB represent a section of the regularly-measured base about equal to the distance to be crossed, with its ends carefully marked. Recommence the measurement on the other side of creek or bog, and mark off a similar distance, CD, all in the line of the base. Set up and adjust the theodolite on firm ground at P and in sight of the stubs at A, B, C, and D. At the intersection of the cross-lines on the tacks at these four points, or in line to P, insert a short wire, or nail, or the smallest object distinctly visible from P. Then measure the angles APD, APC, and APB, and for verification the angle BPD.



Example.—Crossing of Rudie Creek.

Let AB $= a$ $= 90^{m}.0242$ = distance corrected for inclination and temperature.
CD $= b$ $= 120^{m}.0316$ = distance corrected for inclination and temperature.
BC $= x$ = the unmeasured distance.
APB $= A^{1} = 19^{\circ}\ 41'\ 44''.56$ = mean of observations.
APC $= A^{2} = 49^{\circ}\ 02'\ 29''.77$ = mean of observations.
APD $= A^{3} = 75^{\circ}\ 22'\ 02''.56$ = mean of observations.
BPD $= A^{4} = 55^{\circ}\ 40'\ 14''.50$ = mean of observations.

$$x = -\frac{a+b}{2} \pm \frac{a-b}{2\cos y} \qquad \tan^{2} y = \frac{4ab}{(a-b)^{2}} \cdot \frac{\sin A^{2} \sin (A^{3} - A^{1})}{\sin A^{1} \sin (A^{3} - A^{2})}$$

	log's.	
$\sin (A^{3} - A^{1}) = 55^{\circ}\ 40'\ 16''.25$	9.9168826	
$\sin A^{2} = 49^{\circ}\ 02'\ 29''.8$	9.8780538	
$\sin (A^{3} - A^{2}) = 26^{\circ}\ 19'\ 32''.8$ (arc co)	0.3531314	
co. $\sin A^{1} = 19^{\circ}\ 41'\ 44''.6$	0.4723380	
$a = 90^{m}.0242$	1.9543593	
$b = 120^{m}.0316$	2.0792956	
4	0.6020600	
$(a-b)^{2}$ (arc co)	7.0455432	
$\tan^{2} y$	2.3016639	
$\tan y$	1.1508319	
$y = 85^{\circ}\ 57'\ 29''.7$		
$(a-b)$	1.4772284	
$\cos y$ (arc co)	1.1519135	
2 (arc co)	9.6989700	
$\frac{a-b}{2\cos y}$	2.3281119	$212^{m}.8687$
		$\frac{a+b}{2} = 105^{m}.0279$
		$x = 107^{m}.3\ldots$

For verification, change the position of P, and from the new point observe the same angles, and with these recompute x. In the above the unmeasured distance determined from two positions was found to be identical.

Verification of measurement.—Care must be taken, while making the contact, to bring the knife-edge to the center of plane; to hold the magnifying glass parallel with the rod when bringing the line on the index plate and slide to coincide; to make no mistake in reading the thermometer or angle of inclination, and especially in the adjustment of the transit for vertical offsets, or when the end of the rod is to be transferred to a stub.

To test the accuracy of the measurement, a part of the line should be always remeasured, and, if there is any doubt, the entire line. The following comparison is copied from the record of the measurement on the Virginia Coast:

August 30, 1867, from bar 148 to bar 286, inclusive,		
Mean of rods = $6^m.00164579 \times 138$ bars	828.22711902	
Correction for temperature above 75°	+0.00227743	
Correction for inclination of rods	—0.02329017	
		828.20610628
September 2, same distance remeasured,		
Mean of rods = $6^m.00164579 \times 138$ bars	828.22711902	
Correction for temperature 75°	+0.02084498	
Correction for inclination of rods	—0.02279271	
Correction for error in alignment of 19 bars	+0.00007717	
Measured excess, at 286th bar	—0.01970000	
		828.20554846
Difference about two-hundredths of an inch		0.00055600

Length of line.—The mean temperature of the rods during the measurement is obtained by multiplying the number of bars (usually, but not always, ten) between the observed temperatures, by the mean of the four temperatures, and by dividing the sum of these quantities by the total number of bars measured. The corrections for inclination are added, and the sum subtracted from the measured distance.

Rod No. 1 = $6^m.00168541$ at 75° F. × 404 bars	2424.68090564
Rod No. 2 = $6^m.00160616$ at 75° F. × 404 bars	2424.64888864
Distance across Rudie Creek computed	+107.84080000
Distance beyond 826th bar to center of station	+ 0.71930000
Mean temperature = 81°.77, correction for 6°.77 above 75°	+ 0.21094919
Correction for inclination of rods	— 0.16968103
	4957.93116244

When two lines and the included angle are measured:

Let $a = 4957^m.931162$

$b = 4307^m.587099$

C = 177° 37′ 59″.82, or deviation from straight line = 2° 22′ 00″.18

θ = 2° 22′ 01″.69, reduced to angle of chords, or 142′.0282

$$\text{correction} = \frac{\sin^2 1'}{2} \cdot \frac{ab\theta^2}{a+b}$$

log θ	2.1523746	
log θ	2.1523746	
log a	3.6953005	
log b	3.6342341	
co. log $(a+b)$	6.0331303	
constant = $\frac{\sin^2 1'}{2}$ log constant	2.6264222	
	0.2938363	-1.967247
		$a+b = 9265.518261$
Length of line connecting extreme points		9263.551014

Four lines were measured in the fall of 1867, the maximum deviation in direction being 4° 27′ 06″. The correction to the sum of the measured distances to reduce them to a straight line was $32^{m}.016$,

	m
And the resulting length of the direct line	14270.931
Reduction to mean level of sea	— 0.005
	14270.926

To reduce to level of half-tide,

K = the distance or length of line.
h = mean height of bars above half-tide.
R = radius of curvature of the arc.

$$\text{Correction} = \frac{K h}{R}$$

log K = $14270^{m}.926$	4.15445
log h = $2^{m}.36$	0.37291
co. log R	3.19660
correction = $0^{m}.005296$	7.72400

At each station the horizon was closed by observing the supplemental angle, and while occupying the initial point, the azimuth of the triangulation was transferred to the terminus of each of the measured lines, as well as to the most distant visible station, to serve as checks and conditions.

RECORDS, DUPLICATES, AND COMPUTATIONS.

Beside the monthly and annual reports to be made to the Superintendent, in conformity with his "General Instructions," the following records, appertaining to the triangulation, are directed to be forwarded to or deposited at the office:

I. A carefully prepared copy of the original observations.

II. A copy of the descriptions of stations.

III. A copy of the plan of the triangulation, the primary on a scale of $\frac{1}{400000}$, the secondary $\frac{1}{100000}$, and the tertiary on a scale of $\frac{1}{40000}$. These plans should show each unobserved angle by the usual broken line for half the length of one or both of the enclosing sides; also the general features of the country embraced within its limits; and, in the tertiary, a sketch of the shore-line.

The above duplicates are generally kept up in the field as the work advances, and are forwarded to the office at the close of the season's operation.

IV. The original volumes containing the observations of horizontal and vertical angles.

V. The original volume, entitled "Descriptions of Stations."

VI. An abstract of the measurements and resulting angles, corrected, if necessary, for phase of signals and eccentricity of stations.

VII. A cahier containing the triangle side computations.

VIII. The L. M. Z. computations.

These are all forwarded to the office as soon as possible after the assistant has completed his computations.

NOTE.

As the measurements in the survey of the coast are made and expressed in meters, the following values and logarithms from the latest comparisons, are given to convert the distances, horizontal and vertical, found in the preceding pages, into inches, feet, yards, and miles.

Meters ×	39.370432	= inches, or to log of meters add 1.5951701
Meters ×	3.280869	= feet, or to log of meters add 0.5159889
Meters ×	1.093623	= yards, or to log of meters add 0.0388676
Meters ×	0.000621377	= miles, or to log of meters add 6.7933550

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Methods, discussions, and results.
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